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Combinatorial Aspects of Abstract Young Representations (Extended Abstract)

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Abstract. *The goal of this paper is to give a new unified axiomatic approach to the representation theory of Coxeter groups and their Hecke algebras. Building upon fundamental works by Young and Kazhdan-Lusztig, followed by Vershik and Ram, we propose a direct combinatorial construction, avoiding a priori use of external concepts (such as Young tableaux). This is carried out by a natural assumption on the representation matrices. For simply laced Coxeter groups, this assumption yields explicit simple matrices, generalizing the Young forms. For the symmetric groups the resulting representations are completely classified and include the irreducible ones. Analysis involves generalized descent classes and convexity (à la Tits) within the Hasse diagram of the weak Bruhat poset.*

Résumé. *L'objectif de cet article est de donner une nouvelle approche axiomatique unifiée de la théorie des représentation des groupes de Coxeter et de leurs algèbres de Hecke. En utilisant les travaux de Young, Kazhdan-Lusztig ainsi que de Vershik et Ram, nous proposons une construction combinatoire directe qui évite l'introduction de concepts extérieurs (par exemple les tableaux de Young). Cette construction est faite à partir d'une hypothèse naturelle sur les matrices de représentation. Pour les groupes de Coxeter simplement lacé, cette hypothèse donne des matrices simples explicites, généralisant la forme de Young. Pour les groupes symmétriques les représentations associèes sont complètement classifiées, en particulier celles qui sont irréductibles. Ce travail utilise les classes de descente généralisées et la convexité (à la Tits) dans le diagramme de Hasse de l'ordre de Bruhat faible.*

1. Introduction

The goal of the construction of abstract Young representations, presented in [**ABR1**], is to give a new unified axiomatic approach to the representation theory of Coxeter groups and their Hecke algebras.

We want our construction to

(a) apply in a general context;
(b) be simple, direct and **combinatorial**; and
(c) avoid a priori use of concepts external to the group or algebra itself (such as standard Young tableaux).

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Goals (a) and (c) were stated and pursued by Kazhdan-Lusztig [**KL**] and Vershik [**V**]. Goal (b) was posed in [**BR**].

Our guiding lines are two fundamental methods to construct representations: Young theory and Kazhdan-Lusztig theory.

In *Young Theory* (as explained by James [**J**]) the construction starts with Young tableaux, which are sophisticated ad-hoc combinatorial objects. Modules (in particular, irreducible ones) are generated by symmetrizers of Young tableaux. Representing matrices are obtained as a side benefit.This theory is effective for classical Weyl groups. For a detailed description see [**J**] and [**JK**].

Kazhdan-Lusztig Theory [**KL**] is a very general approach to the construction of Hecke algebra representations. A distinguished *basis*, indexed by group elements, is compatible with the decomposition of the Hecke algebra. The Coxeter group acts on linear spaces with bases indexed by special subsets of the group, called *cells.* The basic tools in this construction are Kazhdan-Lusztig polynomials. Resulting representation matrices are given in terms of coefficients of these polynomials [**Hu2**, §7.14]. Unfortunately, these coefficients (and thus entries of the representing matrices) are very difficult to compute. For an axiomatic approach to this construction via cellular algebras see [**GL**].

The idea of "reversing Young theory", namely, constructing representations using explicit representation matrices for the Coxeter generators, is apparently due to Vershik [**V**], and was further developed in works of Vershik-Okounkov [**V**] [**OV**], Pushkarev [**P**] and Ram [**Ra1**] (see also [**BR**]). In these papers the external objects (Young tableaux, or abstractions thereof) are applied as an important initial ingredient.

Our approach is different. The idea is, again, to reverse Young theory — but along "Kazhdan-Lusztig language". As in Kazhdan-Lusztig theory, we start with a (formal) *basis* indexed by group elements; decomposition is compatible with special subsets of the group, called *cells.* The action is assumed to satisfy a natural condition, as follows.

Let (W, S) be a Coxeter system, and let $\mathcal{K}$ be a subset of W. Let F be a suitable field of characteristic zero (e.g., the field $\boldsymbol{C}(q)$ in the case of the Iwahori-Hecke algebra of type A), and let ρ be a representation of (the Iwahori-Hecke algebra of) W on the vector space $V_{\mathcal{K}} := span_F\{C_w \mid w \in \mathcal{K}\}$, with basis vectors indexed by elements of $\mathcal{K}$. Motivated by goals (a)–(c) above, we want to study the sets $\mathcal{K}$ and representations ρ which satisfy the following axiom:

(A) *For any generator $s \in S$ and any element $w \in \mathcal{K}$ there exist scalars $a_s(w), b_s(w) \in F$ such that*

$$\rho_s(C_w) = a_s(w)C_w + b_s(w)C_{ws}.$$

If $w \in \mathcal{K}$ but $ws \notin \mathcal{K}$ we assume $b_s(w) = 0$.

A pair $(\rho, \mathcal{K})$ satisfying Axiom (A) is called an *abstract Young (AY) pair;* ρ is an *AY representation,* and $\mathcal{K}$ is an *AY cell.* If $\mathcal{K} \neq \emptyset$ and has no proper subset $\emptyset \subset \mathcal{K}' \subset \mathcal{K}$ such that $V_{\mathcal{K}'}$ is ρ-invariant, then $(\rho, \mathcal{K})$ is called a *minimal AY pair.* (This is much weaker than assuming ρ to be irreducible.)

Surprisingly, Axiom (A) leads to very concrete matrices, whose entries are essentially inverse linear. Analysis of the construction involves a convexity theorem of Tits [**T**] and the generalized descent classes introduced by Björner and Wachs [**BW1**].

This extended abstract is based on the paper [**ABR1**]. Main definitions and results of that paper are surveyed in Sections 2 and 3. A new result on boundary conditions, not yet available in preprint form, is proved in Section 4. A combinatorial characterization of minimal AY cells and representations for the symmetric group is given in Section 5. For proofs and more details see [**ABR1**].

Note Added: Having completed the current version of [**ABR1**], we have been informed of the important recent paper [**Ra2**]. Although it differs from our work in context, initial assumptions, motivation and language, there are points of contact and similarity in some of the results. In particular, the linear functional $\langle f, \cdot \rangle$ which appears in the coefficients of a minimal AY pair (see Theorem 3.4 below) is a basic ingredient in [**Ra2**].

2. Abstract Young Cells

Recall the definition of AY cells and representations from the previous section.

Problem 2.1. (Kazhdan [**K**]) *Given a subset $\mathcal{K} \subseteq W$, how many nonisomorphic abstract Young representations may be defined on $V_{\mathcal{K}}$?*

In particular,

Problem 2.2. *Which subsets of W are (minimal) AY cells?*

Observation 2.3. Every nonempty AY cell is a left translate of an AY cell containing the identity element of W.

Let T be the set of all reflections in W, and let $A \subseteq T$ be any subset. The (left) *A-descent set* of an element $w \in W$ is defined by

$$Des_A(w) := \{t \in A \mid \ell(tw) < \ell(w)\}.$$

For $D \subseteq A \subseteq T$, the corresponding *generalized descent class* is

$$W_A^D := \{w \in W \mid Des_A(w) = D\}.$$

These sets were studied by Tits [**T**, Ch. 2] and Björner-Wachs [**BW1, BW2**].

The *right Cayley graph* $X(W,S)$ has W as the set of vertices, and has a directed edge $w \to ws$ whenever $w \in W$ and $s \in S$. A subset $\mathcal{K}$ of W is *convex* in $X(W,S)$ if every shortest path between any two elements of $\mathcal{K}$ has all its vertices in $\mathcal{K}$.

Using [**T**, Theorem 2.19] we prove

Theorem 2.1. *Every minimal AY cell is a generalized descent class; in particular, it is convex in the right Cayley graph $X(W,S)$ (or, equivalently, under right weak Bruhat order).*

3. Abstract Young Representations

In [**ABR1**] it is shown that, under mild conditions (see Theorem 3.1 below), Axiom (A) is equivalent to the following more specific version.

(B) *There exist scalars $\dot{a}_t, \dot{b}_t, \ddot{a}_t, \ddot{b}_t \in F$ ($\forall t \in T$) such that, for all $s \in S$ and $w \in \mathcal{K}$:*

$$\rho_s(C_w) = \begin{cases} \dot{a}_{wsw^{-1}} C_w + \dot{b}_{wsw^{-1}} C_{ws}, & \text{if } \ell(w) < \ell(ws); \\ \ddot{a}_{wsw^{-1}} C_w + \ddot{b}_{wsw^{-1}} C_{ws}, & \text{if } \ell(w) > \ell(ws). \end{cases}$$

If $w \in \mathcal{K}$ and $ws \notin \mathcal{K}$ we assume that $\dot{b}_{wsw^{-1}} = 0$ (if $\ell(w) < \ell(ws)$) or $\ddot{b}_{wsw^{-1}} = 0$ (if $\ell(w) > \ell(ws)$).

Theorem 3.1. *Let $(\rho, \mathcal{K})$ be a minimal AY pair for the Iwahori-Hecke algebra of (W,S). If $a_s(w) = a_{s'}(w') \Longrightarrow b_s(w) = b_{s'}(w')$ ($\forall s, s' \in S, w, w' \in \mathcal{K}$), then Axioms (A) and (B) are equivalent.*

This theorem shows that the coefficients $a_s(w)$ and $b_s(w)$ in Axiom (A) depend only on the reflection $wsw^{-1} \in T$ and on the relation between w and ws under right weak Bruhat order.

The assumption regarding the coefficients $b_s(w)$ in Theorem 3.1 is merely a normalization condition. Thus, in order to determine an AY representation, it suffices to determine the coefficients $\dot{a}_t$ and $\ddot{a}_t$ (actually, $\dot{a}_t$ will suffice) for all reflections t and to choose a normalization for the $\dot{b}_t$ and $\ddot{b}_t$. One such normalization is defined as follows (assuming, for simplicity, that $\mathcal{K}$ contains the identity element of W). Let

$$\begin{aligned} T_{\mathcal{K}} &:= \{wsw^{-1} \mid s \in S,\, w, ws \in \mathcal{K}\}, \\ T_{\partial\mathcal{K}} &:= \{wsw^{-1} \mid s \in S,\, w \in \mathcal{K},\, ws \notin \mathcal{K}\}. \end{aligned}$$

Fact 3.1.

$$T_{\mathcal{K}} \cap T_{\partial\mathcal{K}} = \emptyset.$$

The *row stochastic* normalization satisfies

$$\dot{a}_t + \ddot{a}_t = 1 - q, \quad \dot{b}_t = 1 - \dot{a}_t, \quad \ddot{b}_t = 1 - \ddot{a}_t \qquad (\forall t \in T_{\mathcal{K}});$$

$$\dot{a}_t \in \{1, -q\}, \quad \dot{b}_t = 0 \qquad (\forall t \in T_{\partial\mathcal{K}}).$$

Problem 3.2. (Kazhdan [**K**]) *Do the coefficients $a_s(w)$ determine all the character values?*

An (affirmative) answer to this problem, independent of the choice of normalization, will be given in [**ABR3**].

It turns out that for simply laced Coxeter groups the coefficients $\dot{a}_t$ are given by a linear functional (see Theorems 3.3 and 3.4 below).

Let V be the root space of W, and let $\langle\ ,\ \rangle$ be an arbitrary positive definite bilinear form on V. For a reflection $t \in T$, let $\alpha_t \in V$ be the corresponding positive root.

Definition 3.2. Let $\mathcal{K}$ be a convex subset of W containing the identity element. A vector f in the root space V is $\mathcal{K}$-generic if:

(i) For all $t \in T_{\mathcal{K}}$,

$$\langle f, \alpha_t \rangle \notin \{0, 1, -1\}.$$

(ii) For all $t \in T_{\partial\mathcal{K}}$,

$$\langle f, \alpha_t \rangle \in \{1, -1\}.$$

(iii) If $w \in \mathcal{K}$, $s, t \in S$, $(st)^3 = 1$ and $ws, wt \notin \mathcal{K}$ then

$$\langle f, \alpha_{wsw^{-1}} \rangle = \langle f, \alpha_{wtw^{-1}} \rangle.$$

By Observation 2.3, every abstract Young representation is isomorphic to one on an AY cell containing the identity element. Therefore, in the following theorems, we assume that $\mathcal{K}$ contains the identity element.

Theorem 3.3. *Let W be an irreducible simply laced Coxeter group, and let $\mathcal{K}$ be a convex subset of W containing the identity element. Let $\langle\ ,\ \rangle$ be an arbitrary positive definite bilinear form on the root space V. If $f \in V$ is $\mathcal{K}$-generic then*

$$\dot{a}_{wsw^{-1}} := \frac{1}{\langle f, \alpha_{wsw^{-1}} \rangle} \qquad (\forall w \in \mathcal{K}, s \in S),$$

together with $\ddot{a}_{wsw^{-1}}$, $\dot{b}_{wsw^{-1}}$ and $\ddot{b}_{wsw^{-1}}$ satisfying appropriate normalization conditions, define a representation ρ such that $(\rho, \mathcal{K})$ is a minimal AY pair.

For $n \in \boldsymbol{Z}$ let

$$[n]_q := \frac{1 - q^n}{1 - q} \in \boldsymbol{Z}[q, q^{-1}].$$

Replacing $\langle f, \alpha_t \rangle$ by its q-analogue $[\langle f, \alpha_t \rangle]_q$ gives representations of the Iwahori-Hecke algebra $\mathcal{H}_q(W)$. See [**ABR1**, Theorem 8.5].

The following theorem is complementary.

Theorem 3.4. *Let W be an irreducible simply laced Coxeter group and let $\mathcal{K}$ be a subset of W containing the identity element. Assume that $\dot{a}_{wsw^{-1}} \neq 0$ ($\forall w \in \mathcal{K}, s \in S$). If $(\rho, \mathcal{K})$ is a minimal AY pair satisfying Axiom (B) then there exists a $\mathcal{K}$-generic $f \in V$ such that*

$$\dot{a}_{wsw^{-1}} = \frac{1}{\langle f, \alpha_{wsw^{-1}} \rangle} \qquad (\forall\ w \in \mathcal{K}, s \in S).$$

For an Iwahori-Hecke algebra analogue see [**ABR1**, Theorem 8.6].

4. Boundary Conditions

In this section it is shown that the action of the group W on the boundary of a cell determines the representation up to isomorphism. As this result is not yet available in preprint form, it is given with a proof.

Definition 4.1. Let W be a finite Coxeter group, and let V be its root space. A *basic (affine) hyperplane* in V has the form

$$H_{t,\varepsilon} := \{f \in V \mid \langle f, \alpha_t \rangle = \varepsilon\},$$

where $t \in T$ and $\varepsilon = \pm 1$.

A *basic (proper) flat* in V is an intersection (other than $\emptyset$ or V) of basic hyperplanes.

For a basic proper flat L, let

$$A = A_L := \{t \in T \mid L \subseteq H_{t,\varepsilon} \text{ for some } \varepsilon = \pm 1\}.$$

Then $\{W_A^D \mid D \subseteq A\}$ (see Section 2 for the definition of W_A^D) is a partition of W into convex subsets, called the *L-partition* of W.

Let f be a $\mathcal{K}$-generic vector in V. Denote by ρ^f the representation determined by f on $\mathcal{K}$ (say with the row stochastic normalization).

Theorem 4.2. *Let W be a finite simply laced Coxeter group. Let L be a basic proper flat in V, and fix some nonempty convex set $\mathcal{K}$ in the L-partition of W. Then, for any two $\mathcal{K}$-generic vectors $f, g \in L$, the representations ρ^f and ρ^g on $\mathcal{K}$ are isomorphic.*

PROOF. Choose $f_0 \in L$, and let $\{f_1, \ldots, f_k\}$ be a basis for the linear subspace $L - f_0$ of V. Each $f \in L$ has a unique expression as

$$f = f_0 + r_1 f_1 + \cdots + r_k f_k,$$

where $r_1, \ldots, r_k \in \boldsymbol{R}$. For any $t \in T_{\mathcal{K}} \cup T_{\partial\mathcal{K}}$, $\langle f, \alpha_t \rangle$ is a linear combination of $1, r_1, \ldots, r_k$, and is nonzero if f is $\mathcal{K}$-generic. Thus, for any $\mathcal{K}$-generic $f \in L$ and any $s \in S$, each entry of the matrix $\rho^f(s)$ is a rational function of $r_1, \ldots, r_k$; and the same holds for each entry of $\rho^f(w)$ ($\forall w \in W$) and for the character values $Tr(\rho^f(w))$. Note that the coefficients of these rational functions (unlike the actual values of $r_1, \ldots, r_k$) do not depend on the choice of $\mathcal{K}$-generic $f \in L$, even though the set of all such f may be disconnected (see example below). By discreteness of character values and continuity in a small neighborhood of a $\mathcal{K}$-generic $f \in L$, each character value is constant in each such neighborhood, and is thus represented by a constant rational function. It is therefore the same for all the $\mathcal{K}$-generic vectors in L, as claimed. □

Example 4.1. Take $W = S_3 = \langle s_1, s_2 \rangle$ (type A_2) and the basic flat $L = \{f \in V \mid \langle f, \alpha_{s_1 s_2 s_1} \rangle = -1\}$. Then $A = \{s_1 s_2 s_1\}$, and we may choose $\mathcal{K} = \{1_W, s_1, s_2\}$. In that case, $T_{\mathcal{K}} = \{s_1, s_2\}$ and $T_{\partial\mathcal{K}} = \{s_1 s_2 s_1\} = A$. L is an affine line in $V \cong \boldsymbol{R}^2$, and the $\mathcal{K}$-generic points in L form five disjoint open intervals (three of them bounded). For any $\mathcal{K}$-generic vector $f \in L$, ρ^f is the 3-dimensional representation isomorphic to the direct sum of the sign representation and the unique irreducible 2-dimensional representation of S_3.

5. The Symmetric Group

5.1. Minimal AY Cells.

The following theorem characterizes the minimal AY cells in the symmetric group S_n.

Theorem 5.1. *Let $\mathcal{K}$ be a nonempty subset of the symmetric group S_n, and let $\sigma \in \mathcal{K}$. Then $\mathcal{K}$ is a minimal AY cell if and only if there exists a standard Young tableau Q such that*

$$\sigma^{-1}\mathcal{K} = \{\pi \in S_n \mid Q^\pi \text{ is standard}\},$$

where Q^π is the tableau obtained from Q by replacing each entry i by $\pi(i)$.

Proof. First observe that any basic proper flat of the symmetric group contains a vector with integer coordinates. Combining this observation with Theorem 4.2 and Observation 2.3 allows one to reduce the discussion to minimal AY cells, containing the identity element, which are determined by integer valued linear functionals. Theorem 5.2 below completes the proof.

□

For a vector $v = (v_1, \ldots, v_n) \in F^n$ denote

$$\Delta\, v := (v_2 - v_1, \ldots, v_n - v_{n-1}) \in F^{n-1}.$$

For a (skew) standard Young tableau T denote $c(k) := j - i$, where k is the entry in row i and column j of T. Call $cont(T) := (c(1), \ldots, c(n))$ the *content vector* of T, and call $\Delta\, cont(T)$ the *derived content vector* (or *axial distance vector*) of T.

Let $w \in W$, and let f be an arbitrary vector in the root space V of W. Let

$$A_f := \{t \in T \,|\, \langle f, \alpha_t \rangle \in \{1, -1\}\},$$

and denote by $\mathcal{K}^f(w)$ the generalized descent class containing w, taken with respect to $A = A_f$. If f is $\mathcal{K}^f(w)$-generic then the corresponding AY representation of W, with any appropriate normalization, will be denoted $\rho^f(w)$.

Theorem 5.2. *Let $f \in V$ have integer coordinates. Then: the cell $\mathcal{K}^f(1_W)$ is a minimal AY cell for $W = S_n$ if and only if there exists a skew standard Young tableau T of size n such that*

$$f = \Delta\, cont(T).$$

Note that the cell $\mathcal{K}^f(1_W)$ is the generalized descent class $W^{\emptyset}_{A_f}$ (see Section 2). The proof of Theorem 5.2 relies on the following lemmas. The proofs of these lemmas are purely combinatorial, see [**ABR1**]. Here α_{ij} is the positive root corresponding to the reflection (transposition) $(i, j) \in S_n$ $(1 \le i < j \le n)$.

Lemma 5.1. *Under the assumptions of Theorem 5.2, if $i < j$ and $\langle f, \alpha_{ij} \rangle \in \{0, 1, -1\}$ then $w^{-1}(i) < w^{-1}(j)$ for all $w \in \mathcal{K}^f(1_W)$.*

Lemma 5.2. *Let $f \in V$ be an arbitrary vector. The cell $\mathcal{K} := \mathcal{K}^f(1_W)$ is a minimal AY cell for $W = S_n$ if and only if, for all $1 \le i < j \le n$:*

$$\langle f, \alpha_{ij} \rangle = 0 \implies \exists\; r_1, r_2 \in [i+1, j-1] \text{ s.t. } \langle f, \alpha_{ir_1} \rangle = -\langle f, \alpha_{ir_2} \rangle = 1. \tag{5.1}$$

Lemma 5.3. *The vector $c = (c_1, \ldots, c_n) \in \boldsymbol{Z}^n$ is a content vector for some skew standard Young tableaux if and only if for all $1 \le i < j \le n$*

$$c_i = c_j \Longrightarrow \exists r_1, r_2 \in [i+1, j-1] \textit{ s.t. } c_{r_1} = c_i + 1 \textit{ and } c_{r_2} = c_i - 1. \tag{5.2}$$

5.2. Minimal AY Representations of S_n. A direct combinatorial bijection between elements of minimal AY cells and standard Young tableaux follows from Theorem 5.2. This is used to prove the following result.

Theorem 5.3. *The minimal AY representations of the symmetric group S_n are exactly the skew representations, i.e., the representations determined by Young symmetrizers of skew shape. In particular, every irreducible representation of S_n may be realized as a minimal abstract Young representation.*

References

[ABR1] R. M. Adin, F. Brenti and Y. Roichman, *A unified construction of Coxeter group representations (I)*, preprint, 2003, <http://arXiv.org/abs/math.RT/0309364>.

[ABR2] R. M. Adin, F. Brenti and Y. Roichman, *A unified construction of Coxeter Group representations (II)*, in preparation.

[ABR3] R. M. Adin, F. Brenti and Y. Roichman, *A unified construction of Coxeter Group representations (III)*, in preparation.

[ABR4] R. M. Adin, F. Brenti and Y. Roichman, *Descent representations and multivariate statistics*, Trans. Amer. Math. Soc., to appear, <http://arXiv.org/abs/math.CO/0112073>.

[BR] H. Barcelo and A. Ram, *Combinatorial representation theory*, in: New Perspectives in Geometric Combinatorics, MSRI Publications, Vol. 38, 1999, 23–90.

[BB] A. Björner and F. Brenti, *Combinatorics of Coxeter Groups*, Springer Verlag, to appear.

[BW1] A. Björner and M. Wachs, *Generalized quotients in Coxeter groups*, Trans. Amer. Math. Soc. **308** (1988), 1–37.

[BW2] A. Björner and M. Wachs, *Permutation statistics and linear extensions of posets*, J. Combin. Theory (Ser. A) **58** (1991), 85–114.

[Bou] N. Bourbaki, *Elements of Mathematics, Lie Groups and Lie Algebras*, Springer-Verlag, Berlin, 2002.

[GL] J. J. Graham and G. I. Lehrer, *Cellular algebras*, Invent. Math. **123** (1996), 1–34.

[Ho] P. N. Hoefsmit, *Representations of Hecke algebras of finite groups with BN-pairs of classical type*, Ph.D. Thesis, Univ. of British Columbia, 1974.

[Hu1] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer-Verlag, New-York, 1972.

[Hu2] J. E. Humphreys, *Reflection Groups and Coxeter Groups*, Cambridge Studies in Adv. Math. 29, Cambridge Univ. Press, Cambridge, 1990.

[J] G. D. James, *The Representation Theory of the Symmetric Groups*, Lecture Notes in Math. 682, Springer-Verlag, 1978.

[JK] G. D. James and A. Kerber, *The Representation Theory of the Symmetric Group*, Encyclopedia of Math. and its Appl. 16, Addison-Wesley, 1981.

[K] D. Kazhdan, personal communication.

[KL] D. Kazhdan and G. Lusztig, *Representations of Coxeter groups and Hecke algebras*, Invent. Math. **53** (1979), 165–184.

[OV] A. Okounkov and A. M. Vershik, *A new approach to representation theory of symmetric groups*, Selecta Math. (N. S.) **2** (1996), 581–605.

[P] I. A. Pushkarev, *On the theory of representations of the wreath products of finite groups and symmetric groups*, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov (POMI) **240** (1997), Teor. Predst. Din. Sist. Komb. i Algoritm. Metody. 2, 229–244, 294–295; translation in J. Math. Sci. (New York) **96** (1999), 3590–3599.

[Ra1] A. Ram, *Seminormal representations of Weyl groups and Iwahori-Hecke algebras*, Proc. London Math. Soc. **75** (1997), 99–133.

[Ra2] A. Ram, *Affine Hecke algebras and generalized standard Young tableaux*, J. Algebra **230** (2003), 367–415.

[Sa] B. E. Sagan, *The Symmetric Group: Representations, Combinatorial Algorithms and Symmetric Functions*, Wadsworth and Brooks/Cole, Monterey, CA, 1991.

[So] L. Solomon, *A decomposition of the group algebra of a finite Coxeter group*, J. Algebra **9** (1968), 220–239.

[St] R. P. Stanley, *Enumerative Combinatorics*, Vol. 2, Cambridge Studies in Adv. Math. 62, Cambridge Univ. Press, Cambridge, 1999.

[T] J. Tits, *Buildings of Spherical Type and Finite BN-Pairs*, Lecture Notes in Math. 386, Springer-Verlag, Berlin and New-York, 1974.

[V] A. Vershik, *Local algebras and a new version of Young's orthogonal form*, in: Topics in Algebra, Banach Center Publications, Vol. 26 Part 2, PWN-Polish Scientific Publications, Warsaw 1990, 467–473.

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