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Alexander Duality in Combinatorics

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Alexander duality theorem plays a vital role in [**7**] to show that the second Betti number of the minimal graded resolution of the Stanley–Reisner ring $K[\Delta]$ of a simplicial complex Δ is independent of the base field K. On the other hand, a beautiful theorem by Eagon and Reiner [**2**] guarantees that the Stanley–Reisner ideal I_Δ of Δ has a linear resolution if and only if the Alexander dual $\Delta^\vee$ of Δ is Cohen–Macaulay.

With a survey of the recent papers [**3**], [**4**], [**5**] and [**6**], my talk will demonstrate how Alexander duality is used in algebraic combinatorics. More precisely,

- Let $\mathcal{L}$ be a finite meet-semilattice, P the set of join-irreducible elements of $\mathcal{L}$, and $K[\{x_q, y_q\}_{q \in P}]$ the polynomial ring over a field K. We associate each $\alpha \in \mathcal{L}$ with the squarefree monomial $u_\alpha = \prod_{q \leq \alpha} x_q \prod_{q \not\leq \alpha} y_q$. Let $\Delta_{\mathcal{L}}$ denote the simplicial complex on $\{x_q, y_q\}_{q \in P}$ whose Stanley–Reisner ideal is generated by those monomials u_α with $\alpha \in \mathcal{L}$. In the former part of my talk, combinatorics and algebra on the Alexander dual $\Delta_{\mathcal{L}}^\vee$ of $\Delta_{\mathcal{L}}$ will be discussed.
- One of the fascinating results in classical graph theory is Dirac's theorem on chordal graphs ([**1**]). In the latter part of my talk, it will be shown that, via Hilbert–Burch theorem together with Eagon–Reiner theorem, Alexander duality naturally yields a new and algebraic proof of Dirac's theorem.

No special knowledge is required to enjoy my talk.

References

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[4] J. Herzog and T. Hibi, Level rings arising from meet-distributive meet-semilattices, preprint, March 2004, math.AC/0403534.
[5] J. Herzog, T. Hibi and X. Zheng, Dirac's theorem on chordal graphs and Alexander duality, *Europ. J. Combin.*, to appear, math.AC/0307224.
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