Partially directed walks in wedges

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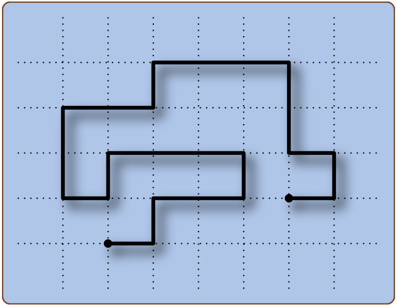
Self-avoiding walks

Self-avoiding walk

- A path on a lattice that does not intersect itself
- $c_n = |\{\text{SAWs of } n \text{ steps}\}|$



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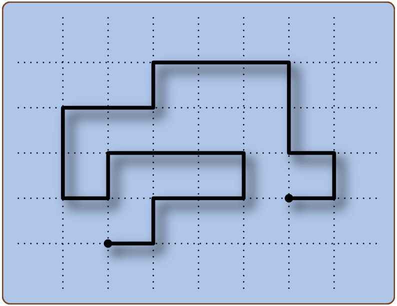

- Computing c_n is a very hard combinatorial problem
- Canonical model of linear polymer in solution

Critical exponents

Scaling of self-avoiding walks

The number of self-avoiding walks grows as

$$c_n \sim A\,\mu^n\,n^{\gamma-1}\,(1+\cdots)$$

- growth constant $\mu_\Box = 2.63815852927(1)$ [Guttmann & Jensen]
- critical exponent $\gamma = 43/32$ [Nienhuis]

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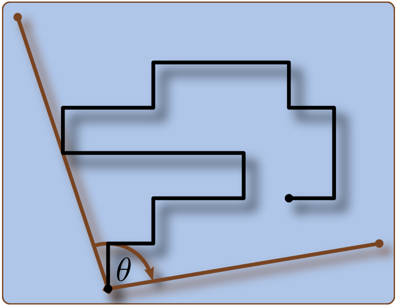
  — conformal field theory
- Much of what is known for 2D lattice models comes from CFT

Put things in wedges



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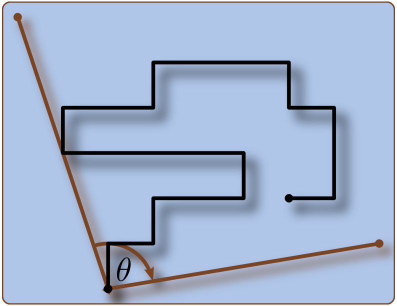

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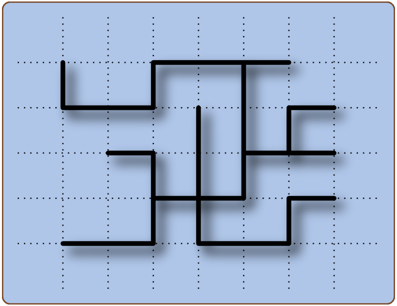
CFT exponent prediction

$$\gamma = 1 + \frac{27}{64} - \frac{15}{32}\frac{\pi}{\theta}$$ [Duplantier & Saleur]

Not conformally invariant

- Many interesting models are not conformally invariant — bond trees



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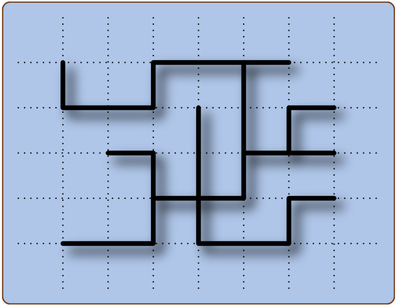

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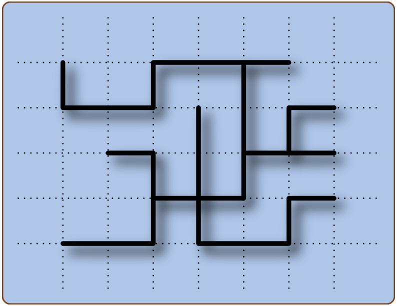

- Much less is known
  — mostly numerical results by series analysis and Monte-Carlo
- Computer enumeration in wedge is hard
  — big growth constant and no FLM

Simulate trees

First problem as a postdoc

- Design algorithm to simulate trees in wedges
- Estimate growth constant λ and critical exponent γ

$$t_n \sim A\,\lambda^n\, n^{\gamma-1}\,(1 + \cdots)$$

- Repeat in different wedges
- Compare and contrast to conformally invariant models — SAWs

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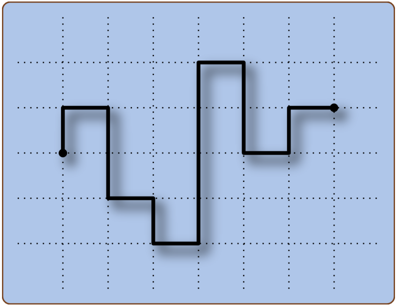
- A few years later we had a more combinatorial idea. . .

Directed paths



Partially directed self-avoiding walk

- A SAW that cannot step west (and ends with an east step)
- Not conformally invariant — behaviour in wedges = ?

Generating function and asymptotics of walks in the plane

- Simple rational generating function

$$P(z) = \sum_{\varphi \in \text{PDSAW}} z^{|\varphi|} = \sum_n p_n z^n = \frac{z(1-z)}{1-2z-z^2}$$

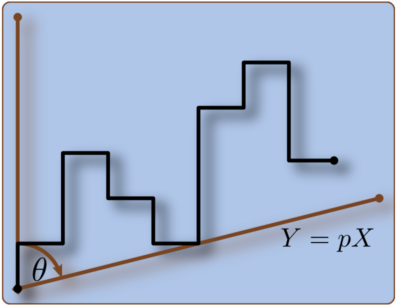
- Dominant singularity gives asymptotics

$$p_n = \frac{\sqrt{2}-1}{2}\left(1+\sqrt{2}\right)^n + o(1)$$

- Growth constant $\mu = 1 + \sqrt{2}$
- Critical exponent $\gamma = 1$

Put them in wedges

- Put PDSAW in an upper wedge — $Y = pX$, $X = 0$



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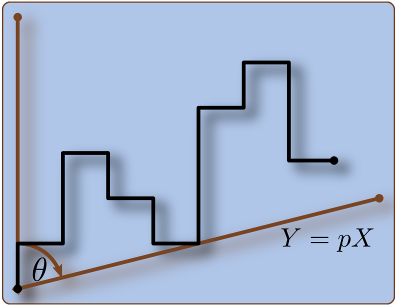
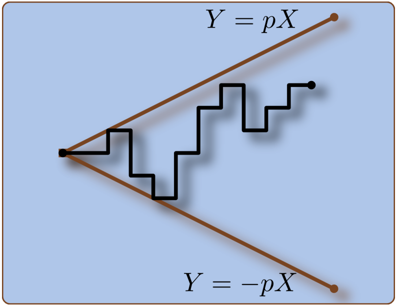


PDSAW in upper wedge

- If $p \in \mathbb{Q}$ then g.f. is algebraic
- Growth constant varies with p (and so θ) [JvR & Ye]

Put them in wedges again

- The symmetric wedge is more interesting — $Y = \pm pX$



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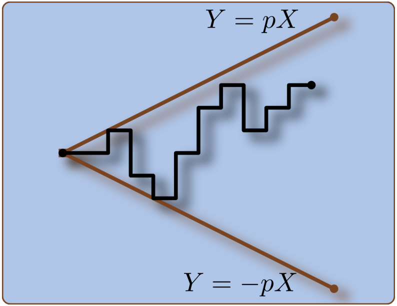

PDSAW in symmetric wedge

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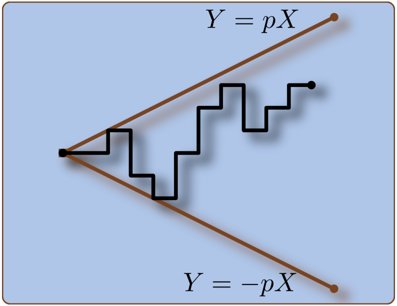

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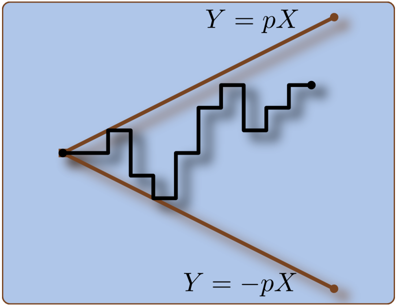


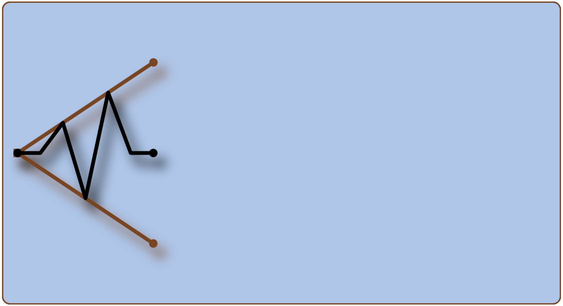
PDSAW in symmetric wedge

- For $p \geq 1$ the growth constant is $1 + \sqrt{2}$

- But what is the critical exponent?
- Need to find g.f. — use the Temperley method

Column-column construction

Each PDSAW in the $Y = \pm pX$ wedge is



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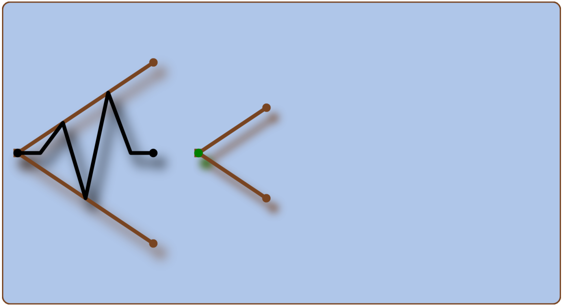

- either a single vertex

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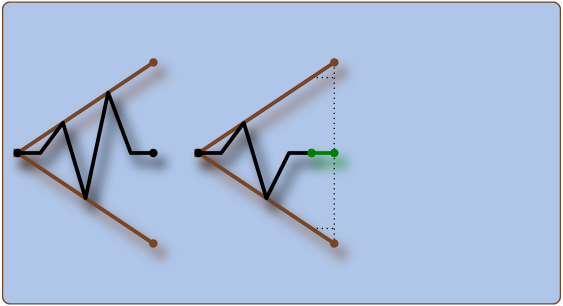

- or obtained by adding an east step

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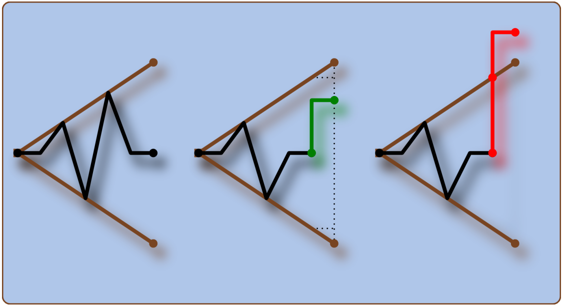

- or by adding north steps and an east step — but not too many north steps

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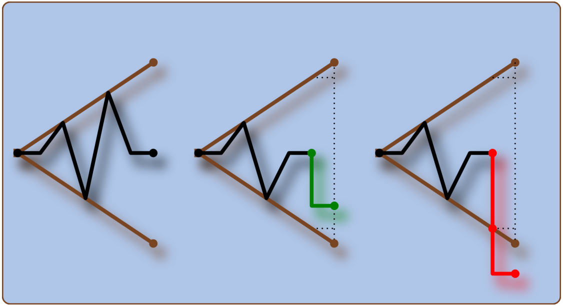

- or obtained by adding south steps and an east step — but not too many south steps

Functional equation

Functional equation in the $Y = \pm pX$ wedge

$$
\begin{aligned}
f_p(a,b) = 1 &+ x(ab)^p f_p(a,b) \\
&+ x(ab)^p \frac{ya/b}{1-ya/b}\Big(f_p(a,b) - f_p(a,ay)\Big) \\
&+ x(ab)^p \frac{yb/a}{1-yb/a}\Big(f_p(a,b) - f_p(by,b)\Big)
\end{aligned}
$$

- $f_p(a,b)$ is the g.f. of PDSAW in this wedge
- x and y are conjugate to # horizontal and # vertical steps
- a and b are conjugate to distance of endpoint from walls

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- Very little progress except for $p = 1$

$p = 1$: solve using the iterated kernel method

Equation for $Y = \pm X$ wedge

$$f(a,b) = 1 + xabf(a,b) + \frac{xya^2}{1 - ya/b}\Big(f(a,b) - f(a,ay)\Big) + \frac{xyb^2}{1 - yb/a}\Big(f(a,b) - f(by,b)\Big)$$

- 1 equation with 3 unknowns

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- 1 equation with 3 unknowns
- Singular when $a = by$ or $b = ay$

Kernelise. . .

Equation for $Y = \pm X$ wedge

$$f(a,b)K(a,b) = X(a,b) + Y(a,b)\,f(a,ay) + Z(a,b)\,f(by,b)$$

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- Symmetry implies

$$f(a,b) = f(b,a) \qquad K(a,b) = K(b,a)$$
$$X(a,b) = X(b,a) \qquad Y(a,b) = Z(b,a)$$

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Equation for $Y = \pm X$ wedge

$$f(a,b)K(a,b) = X(a,b) + Y(a,b)\,f(a,ay) + Y(b,a)\,f(b,by)$$

The kernel

$$K(a,b) = (b-ya)(a-yb)(1-xab) - xyab(a^2+b^2-2yab)$$

- Find the roots of the kernel $b = \beta_{\pm 1}(a)$
- The kernel and $f(a,b)$ can be removed by setting $b = \beta_{+1}(a)$

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$$0 = X\,(a, \beta(a)) + Y\,(a, \beta(a))\;f(a, ay) + Y\,(\beta(a), a)\;f\,(\beta(a), \beta(a)y)$$

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$$f(a,ay) = -\frac{X\left(a,\beta(a)\right)}{Y\left(a,\beta(a)\right)} - \frac{Y\left(\beta(a),a\right)}{Y\left(a,\beta(a)\right)}\,f\left(\beta(a),\beta(a)y\right)$$

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$$\underbrace{f(a,ay)}_{F(a)} = -\frac{X(a,\beta(a))}{Y(a,\beta(a))} - \frac{Y(\beta(a),a)}{Y(a,\beta(a))}\underbrace{f(\beta(a),\beta(a)y)}_{F(\beta(a))}$$

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Now we can iterate. . .

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Formal solution

Back-substitution of first N equations

$$F(a) = \sum_{n=0}^{N} \mathcal{X}(\beta_n) \prod_{k=0}^{n-1} \mathcal{Y}(\beta_k) + F(\beta_{N+1}) \prod_{k=0}^{N} \mathcal{Y}(\beta_k)$$

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- Is this helpful? — mess of nested radicals

Can we simplify this mess?

- At first sight $\beta_n(a)$ is very complicated

$$\beta_{\pm 1}(a) = \frac{a}{2}\left(\frac{1 + y^2 \mp \sqrt{(1-y^2)(1-4xya^2-y^2)}}{y + xa^2 - xy^2a^2}\right)$$

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- Roots of quadratic: $\dfrac{1}{\beta_1(a)} + \dfrac{1}{\beta_{-1}(a)} = \dfrac{1+y^2}{y}\dfrac{1}{a}$
- Substitute $a \mapsto \beta_n$: $\dfrac{1}{\beta_{n+1}} + \dfrac{1}{\beta_{n-1}} = \dfrac{1+y^2}{y}\dfrac{1}{\beta_n}$

Closed form for $\beta_n(a)$

Closed form solution to recurrence

$$\frac{1}{\beta_n(a)} = \frac{y(1-y^{2n})}{y^n(1-y^2)}\frac{1}{\beta_1} - \frac{y^2(1-y^{2n-2})}{y^n(1-y^2)}\frac{1}{a}$$

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Big simplifications

$$f(a, ay) = \sum_{n\geq 0} \mathcal{X}(\beta_n) \prod_{k=0}^{n-1} \mathcal{Y}(\beta_k)$$

Closed form for $\beta_n(a)$

Closed form solution to recurrence

$$\frac{1}{\beta_n(a)} = \frac{y(1-y^{2n})}{y^n(1-y^2)}\frac{1}{\beta_1} - \frac{y^2(1-y^{2n-2})}{y^n(1-y^2)}\frac{1}{a}$$

Big simplifications

$$f(a,ay) = \left(1 + \frac{Q(a)}{y}\right)\sum_{n=0}^{\infty}(-1)^n y^{n^2} Q(a)^n$$

$$Q(a) = \left(\frac{1}{xa^2} - \frac{y}{xa\beta_1} - y\right)$$

Generating function

Generating function of PDSAW in $Y = \pm X$ wedge

- Put $x = t$ and $y = t$:

$$f(1,1) = \frac{1-t}{1-2t-t^2} - \frac{1-t^2-\sqrt{(1-t^2)(1-5t^2)}}{1-2t-t^2} \sum_{n=0}^{\infty} (-1)^n t^{n^2} Q(1)^n$$

$$Q(1) = \left(1-3t^2-\sqrt{(1-t^2)(1-5t^2)}\right)/2t$$

Asymptotics when $p = 1$

PDSAW in $Y = \pm X$ wedge

The number of PDSAW of length n, $v_n^{(1)}$, in this wedge grows as

$$v_n^{(1)} = A_0 \left(1+\sqrt{2}\right)^n + \frac{5^{n/2}}{(n+1)^{3/2}} \Big(A_1 + (-1)^n A_2 + O(1/n)\Big)$$

where the constants are

$$A_0 = 0.277309853486031\ldots$$

$$A_1 = 3.714104865336623\ldots \qquad A_2 = 0.206979970208041\ldots$$

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PDSAW in $Y = 0$, $Y = pX$ wedge

For any $1 \le p < \infty$

$$0.2773\ldots \le \lim_{n\to\infty} \frac{v_n^{(p)}}{(1+\sqrt{2})^n} \le (1+\sqrt{2})/2 = 1.2071\ldots$$

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- All scale like PDSAW in the plane

Conclusions

- We have derived functional equations for PDSAW in symmetric wedges
- For the $Y = \pm X$ wedge we can find the g.f.
- We use this to compute asymptotics
- Growth constant and critical exponent are independent of the wedge angle — very different to conformally invariant models

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- For the $Y = \pm X$ wedge we can find the g.f.
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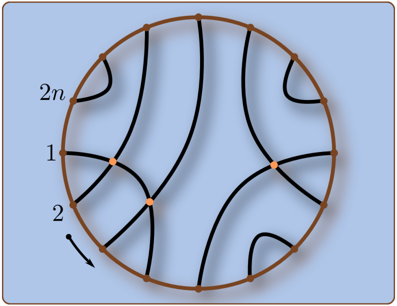
- Growth constant and critical exponent are independent of the wedge angle — very different to conformally invariant models
- Link between PDSAW in wedge and involutions without fixed points

Aside to chord diagrams

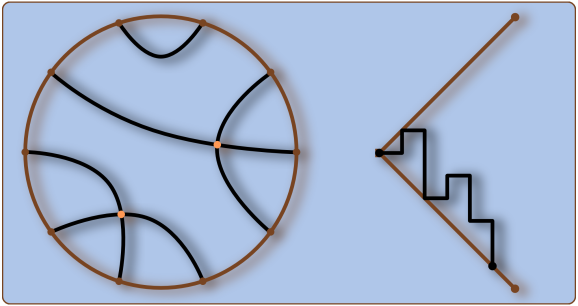


Chord diagrams ≡ involutions without fixed-points

- g.f. of diagrams with n chords in which q counts crossings [Touchard]

$$\Phi_n(q) = \frac{1}{(1-q)^n} \sum_{k=-n}^{n} (-1)^k \binom{2n}{n+k} q^{\binom{k}{2}}$$

Equinumerous — no bijection yet.



PDSAW in wedges and chord diagrams

The number of chord diagrams with n chords and m crossings

$$||$$

the number of PDSAW in the $Y = \pm X$ wedge with n horizontal edges, $n + 2m$ vertical edges and ending at $(n, -n)$