Dual equivalence graphs, ribbon tableaux and Macdonald polynomials

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FPSAC'07, Tianjin, P.R. China

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Macdonald polynomials

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The *transformed Macdonald polynomials* $\widetilde{H}_\mu(x;q,t)$ are the unique functions satisfying the following conditions:

(i) $\widetilde{H}_\mu(x;q,t) \in \mathbb{Q}(q,t)\{s_\lambda[X/(1-q)] \ : \ \lambda \geq \mu\}$,

(ii) $\widetilde{H}_\mu(x;q,t) \in \mathbb{Q}(q,t)\{s_\lambda[X/(1-t)] \ : \ \lambda \geq \mu'\}$,

(iii) $\widetilde{H}_\mu[1;q,t] = 1$.

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(iii) $\widetilde{H}_\mu[1;q,t] = 1$.

The *Kostka-Macdonald polynomials* $\widetilde{K}_{\lambda,\mu}(q,t)$ give the Schur expansion for Macdonald polynomials, i.e.

$$\widetilde{H}_\mu(x;q,t) \;=\; \sum_\lambda \widetilde{K}_{\lambda,\mu}(q,t) s_\lambda(x).$$

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Problem: Find a *combinatorial* proof of positivity.

Better yet, find a combinatorial formula for $\widetilde{K}_{\lambda,\mu}(q,t)$.

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Theorem. (Haglund, Haiman, Loehr 2005)

$$\widetilde{H}_\mu(x;q,t) \;=\; \sum_{S:\mu\rightarrow\mathbb{N}} q^{\mathrm{inv}(S)} t^{\mathrm{maj}(S)} x^S$$

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The Schur functions may be defined by

$$s_\lambda(x) \;=\; \sum_{T\in\mathrm{SSYT}(\lambda)} x^T$$

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Quasi-symmetric functions

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For $\sigma \in \{\pm 1\}^{n-1}$, define the *quasi-symmetric function*

$$Q_\sigma(x) \;=\; \sum_{\substack{i_1 \leq \cdots \leq i_n \\ i_j = i_{j+1} \Rightarrow \sigma_j = +1}} x_{i_1} \cdots x_{i_n}.$$

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Define the *descent signature* $\sigma : \mathrm{SYT} \rightarrow \{\pm 1\}^{n-1}$ by

$$\sigma(T)_i \;=\; \begin{cases} +1 & i \text{ left of } i{+}1 \text{ in } w(T) \\ -1 & i{+}1 \text{ left of } i \text{ in } w(T) \end{cases}$$

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define a vertex-signed graph $\mathcal{G}$ with vertex set V and signature function $\sigma : V \rightarrow \{\pm 1\}^{n-1}$.

Goal: Give sufficient conditions for a vertex-signed graph $\mathcal{G} = (V, \sigma, E)$ to have connected components which satisfy

$$\sum_{v \in \mathcal{C}} Q_{\sigma(v)}(x) \;=\; s_\lambda(x).$$

Dual equivalence and graphs

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For $T, U \in \mathrm{SYT}$, connect T and U with an i-colored edge whenever $w(T)$ and $w(U)$ differ by an elementary dual equivalence for $i-1, i, i+1$.

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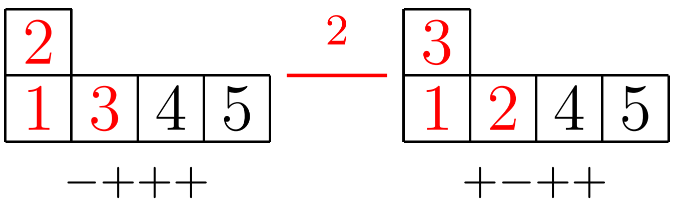

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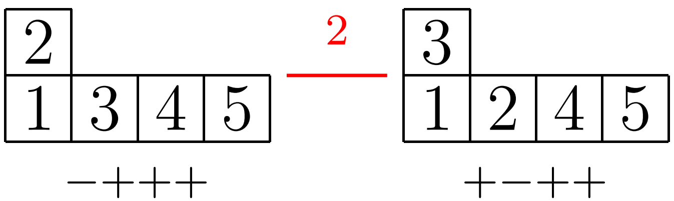

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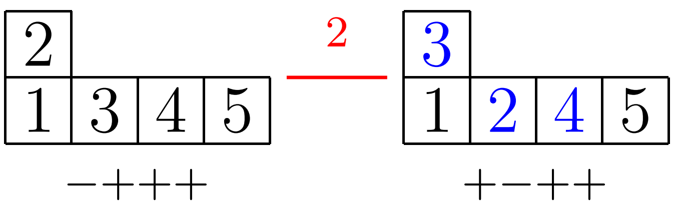

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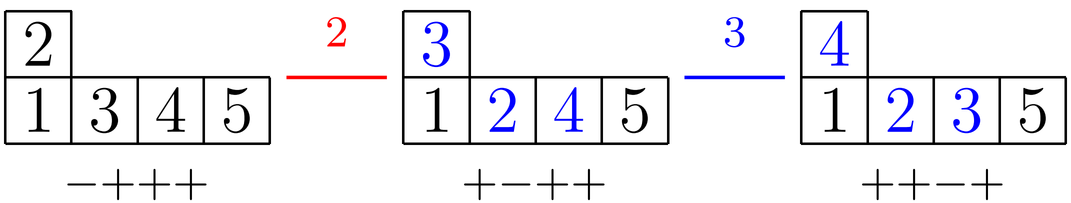

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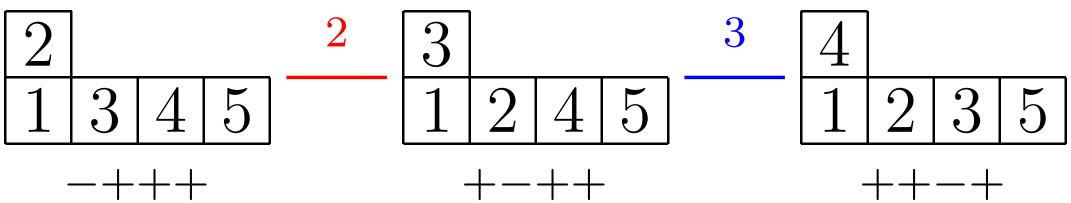

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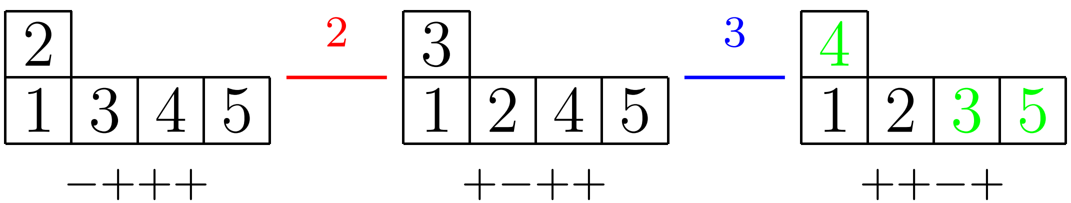

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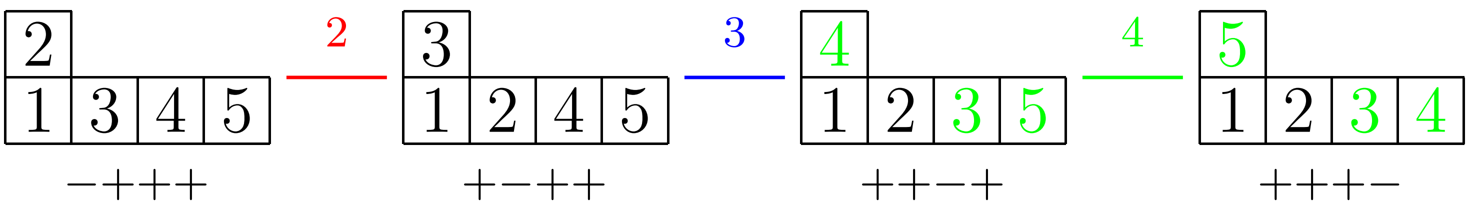

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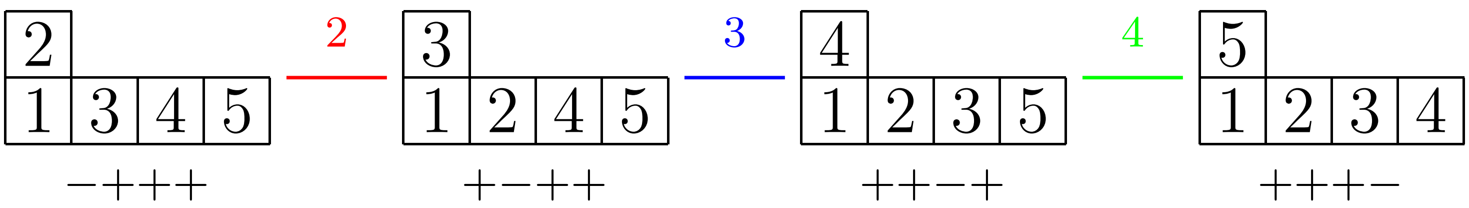

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Goal: Give sufficient conditions for a vertex-signed, edge-colored graph $\mathcal{G} = (V, \sigma, E)$ to have connected components isomorphic to $\mathcal{G}_\lambda$.

Examples of $\mathcal{G}_\lambda$

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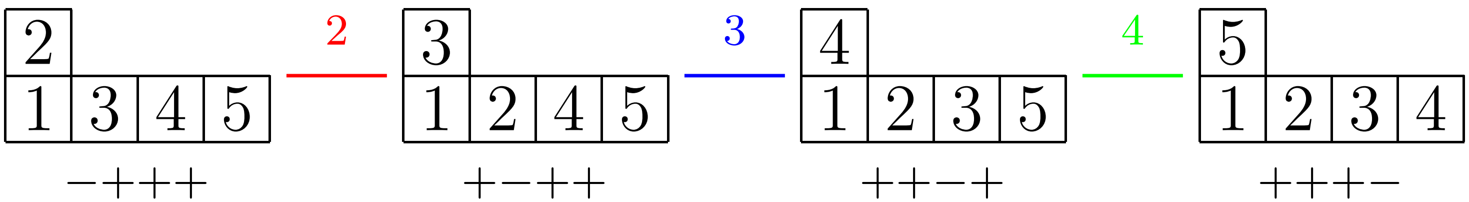

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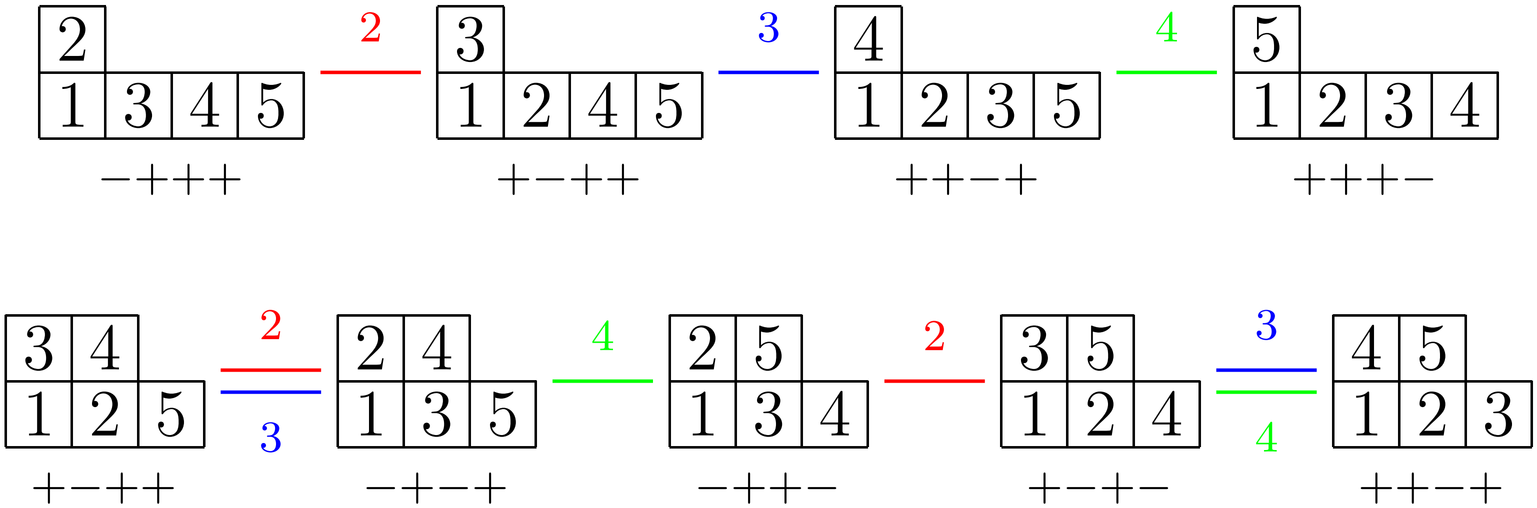

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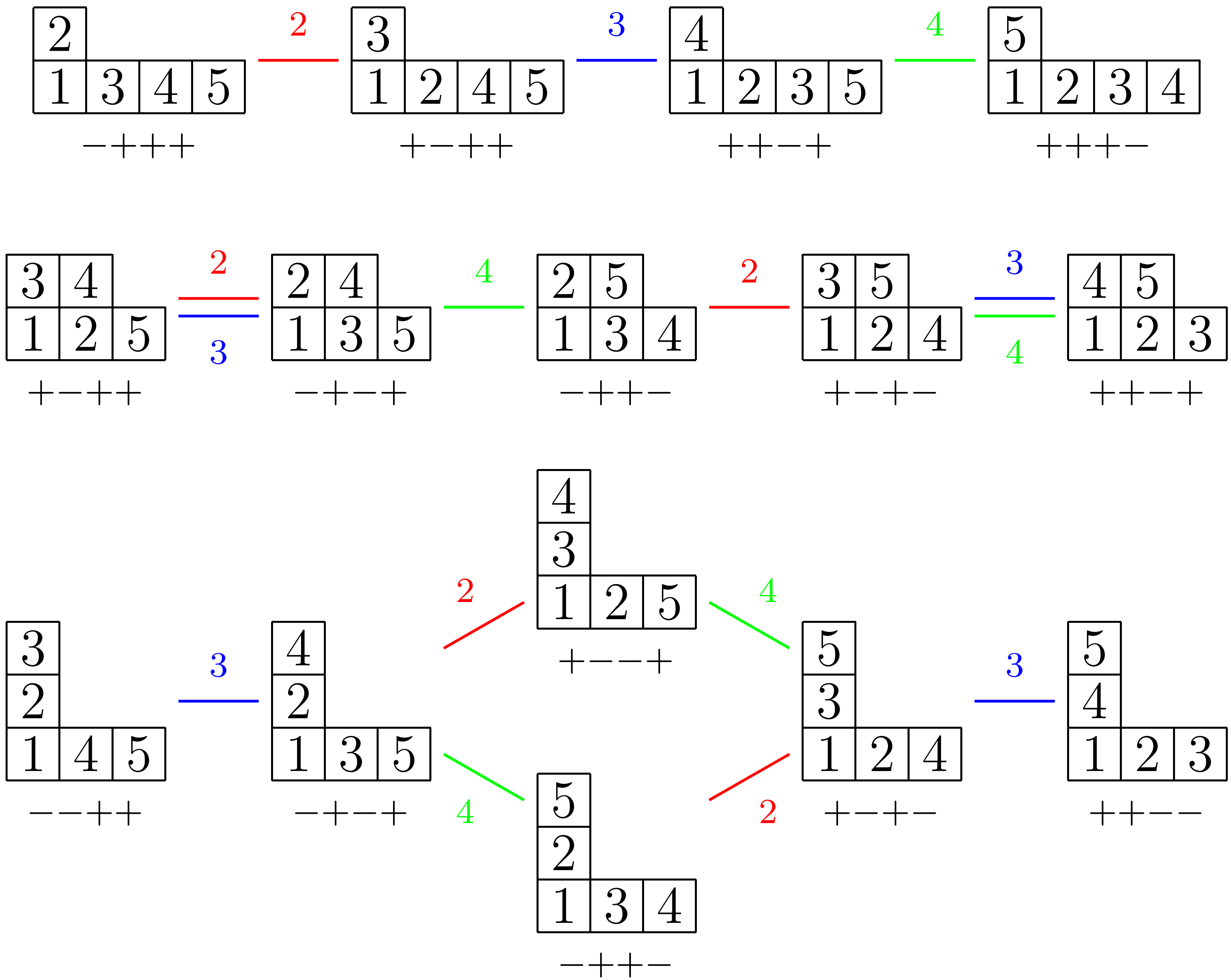

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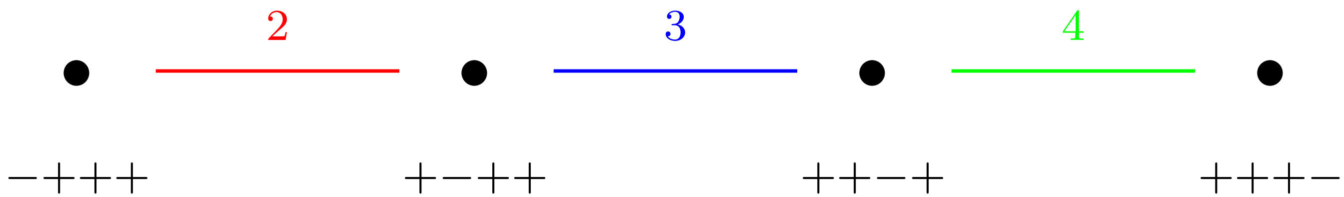

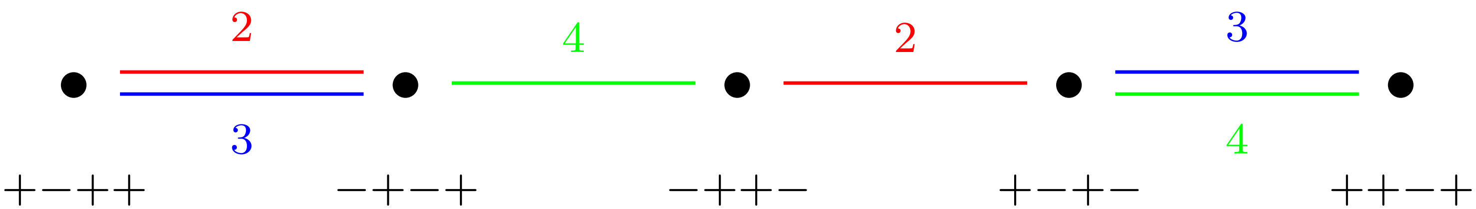

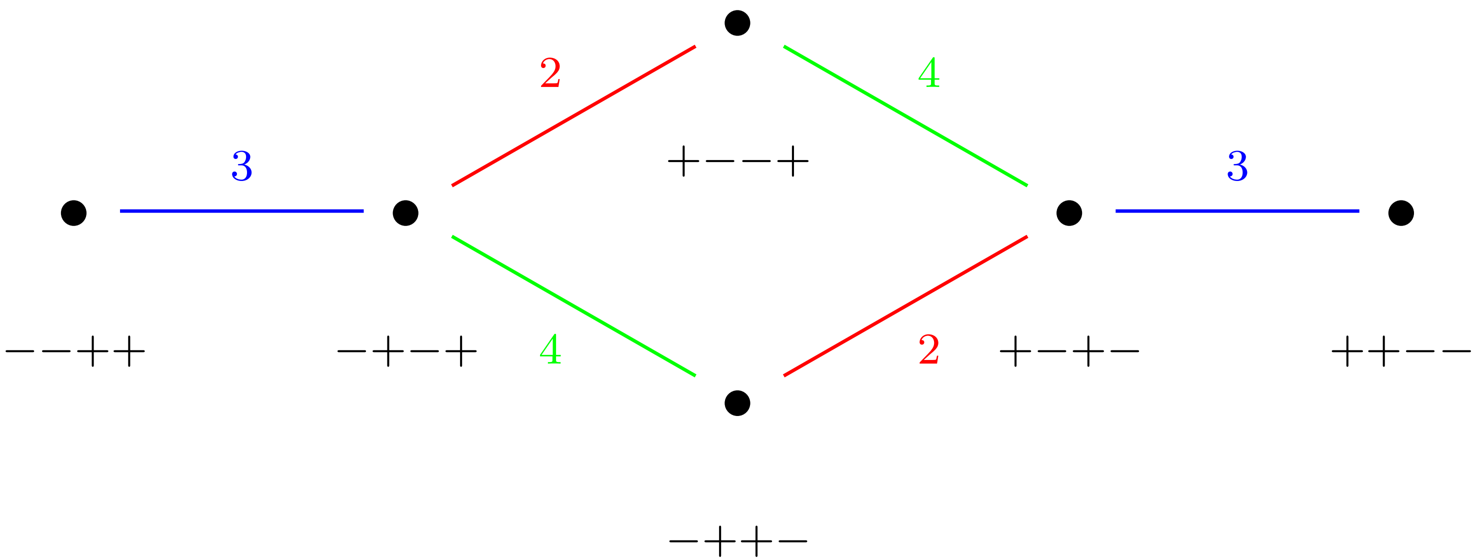

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Theorem. (A.) Every connected component of a DEG is isomorphic to $\mathcal{G}_\lambda$ for a unique partition λ.

Corollary. (A.) If $\mathcal{G}$ is a DEG and α, β are statistics on $V(\mathcal{G})$ which are *constant on connected components*, then

$$\sum_{v \in V(\mathcal{G})} q^{\alpha(v)} t^{\beta(v)} Q_{\sigma(v)}(x) = \sum_{\lambda} \left(\sum_{\mathcal{C} \cong \mathcal{G}_\lambda} q^{\alpha(\mathcal{C})} t^{\beta(\mathcal{C})} \right) s_{\lambda(x)}.$$

Back to Macdonald polynomials

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The inv and maj statistics

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$$D_i^{(k)}(r) = \begin{cases} d_i(r) & \text{if } \operatorname{dist}(i-1,i,i+1) > k \\ \widetilde{d}_i(r) & \text{if } \operatorname{dist}(i-1,i,i+1) \leq k \end{cases}$$

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Making $\mathcal{H}_\mu$ into a DEG

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define i-colored edges by

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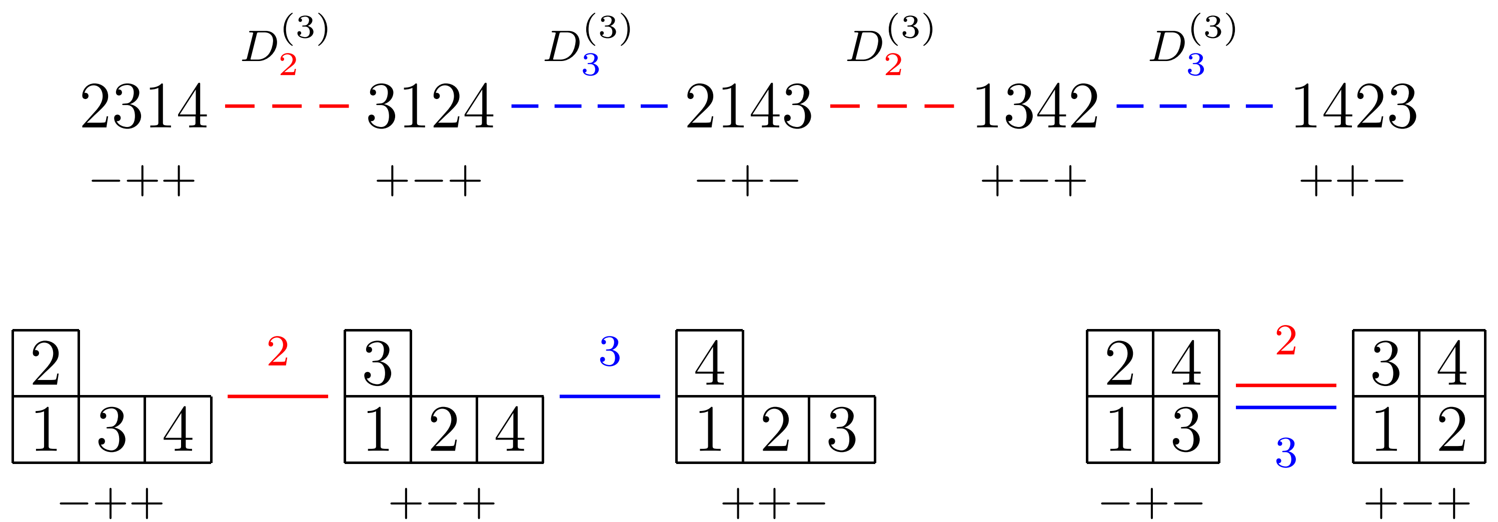

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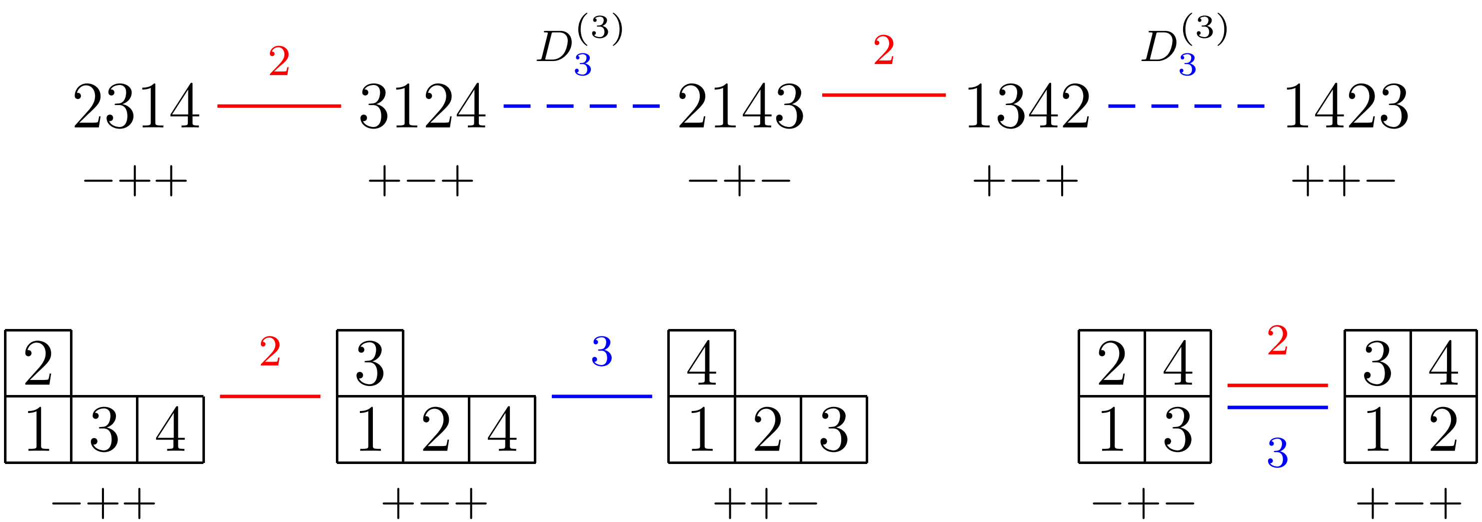

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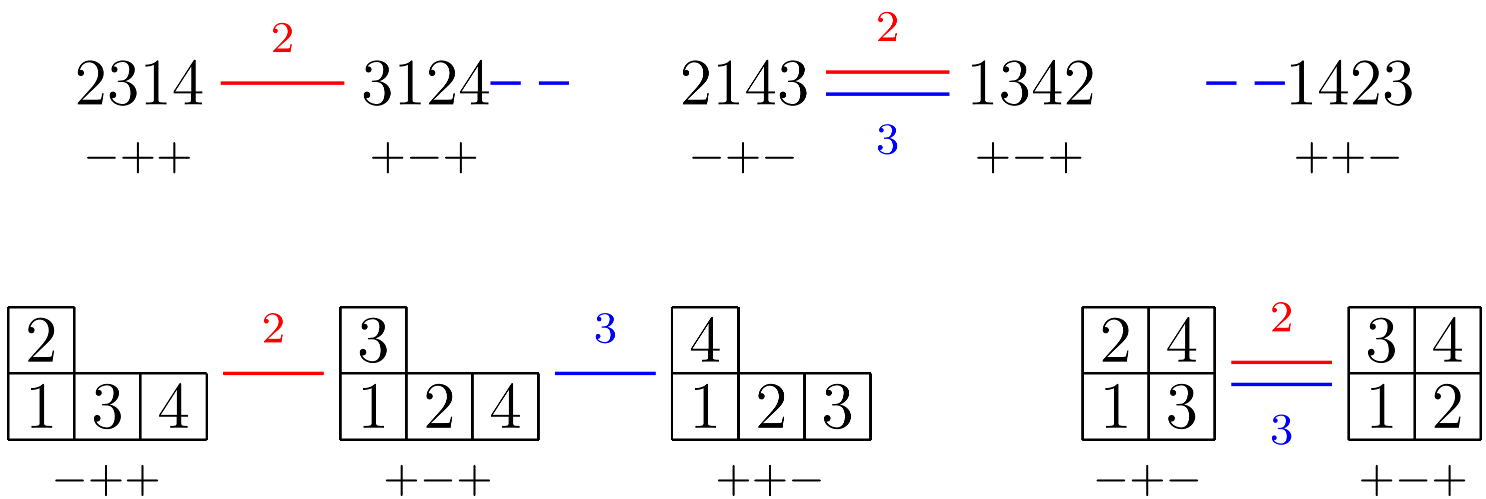

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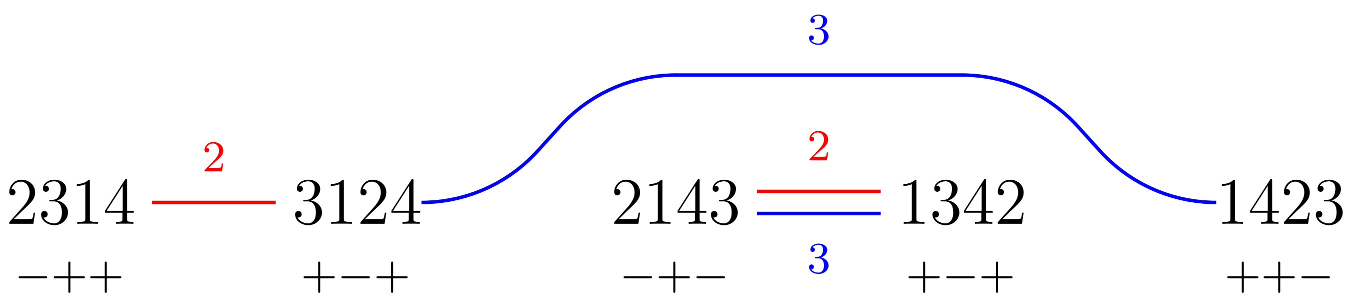

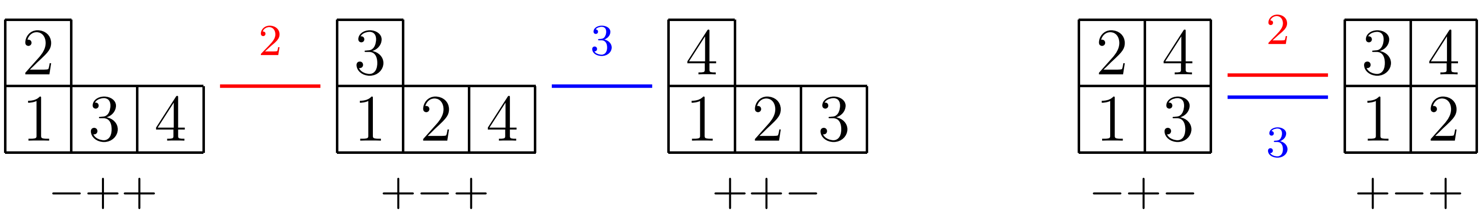

Back to Macdonald positivity

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