Tabloids and Weighted Sums of Characters of Certain Modules of the Symmetric Groups

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FPSAC '07

Coincidence of dimensions

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For some l, $\dim R(0;l) = \dim R(1;l) = \dim R(2;l) = \cdots$.

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We can prove it by $(H, \{ z(i;l) \})$ providing the picture.

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$$\dim R_\mu(0;l) = \dim R_\mu(1;l) = \dim R_\mu(2;l) = \cdots ,$$

where $R_\mu(k;l) = \bigoplus_i R_\mu^{il+k}$.

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where ζ_l is the primitive root of unity.

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  - To show the fact, they explicitly calculated values $Q^\mu_\rho(\zeta^i_l)$ of Green polynomial at roots of unity.

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By their result,

- $Q^{\mu}_{\rho}(\zeta^i_l)$ is a nonnegative integer.

So we have the following questions:

- Does the number have a combinatorial presentation? (Describe as the number of some combinatorial objects.)
- What do the combinatorial objects mean?

Notation

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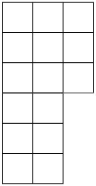
  - Let $\mu = (3^3, 2^3) =$ .

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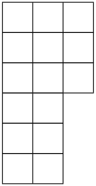
  - Let $\mu = (3^3, 2^3) =$  .
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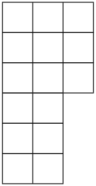
  - Let $\mu = (3^3, 2^3) =$ .
- ρ: a partition of m.
  - Let $\rho = (6, 6, 3) \vdash 15$.

(ρ, l)-tableaux on μ

A (ρ, l)-tableau on μ is a filling satisfying:

- weakly increasing for each row.
- the number of boxes where i lies is ρ_i.
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Example

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$$\begin{array}{|c|c|c|}\hline 1^* & 1 & 3^* \\ \hline 1 & 1 & 3 \\ \hline 1 & 1 & 3 \\ \hline 2^* & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|}\hline 1 & 1 & 3^* \\ \hline 1^* & 1 & 3 \\ \hline 1 & 1 & 3 \\ \hline 2^* & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|}\hline 1 & 1 & 3^* \\ \hline 1 & 1 & 3 \\ \hline 1^* & 1 & 3 \\ \hline 2^* & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|}\hline 1^* & 1 & 3^* \\ \hline 1 & 1 & 3 \\ \hline 1 & 1 & 3 \\ \hline 2 & 2 \\ \cline{1-2} 2^* & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|}\hline 1 & 1 & 3^* \\ \hline 1^* & 1 & 3 \\ \hline 1 & 1 & 3 \\ \hline 2 & 2 \\ \cline{1-2} 2^* & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|}\hline 1 & 1 & 3^* \\ \hline 1 & 1 & 3 \\ \hline 1^* & 1 & 3 \\ \hline 2 & 2 \\ \cline{1-2} 2^* & 2 \\ \cline{1-2} 2 & 2 \\ \cline{1-2}\end{array}, \ldots$$

$$(\mu = (3^3, 2^3),\ \rho = (6, 6, 3),\ l = 3)$$

marked (ρ, l)-tableaux on μ

A marked (ρ, l)-tableau on μ is a diagram obtained

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Main results

Theorem

For $\sigma \in S_m$ of cycle type ρ,

$$\sum_{k \in \mathbb{Z}/l\mathbb{Z}} \zeta_l^k \operatorname{Char}\left(R_\mu(k;l)\right)(\sigma) = \left|\left\{ \text{ marked } (\rho, l)\text{-tableaux on } \mu \right\}\right| .$$

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Remark

$$Q_\rho^\mu(\zeta_l) = \sum_{k \in \mathbb{Z}/l\mathbb{Z}} \zeta_l^k \operatorname{Char}\left(R_\mu(k;l)\right)(\sigma)$$

Key Lemma

Lemma

For an integer j and $\sigma \in S_m$ of cycle type ρ,

$$\sum_{k \in \mathbb{Z}/l\mathbb{Z}} \zeta_l^k \, \mathrm{Char}\left(R_\mu(k;l)\right)(\sigma) = \left| \left\{ \{T\}\text{: a tabloid on } \mu \,\middle|\, \sigma\{T\} = \{T\} a_{\mu,l}^{-j} \right\} \right|.$$

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Lemma

For an integer j and $\sigma \in S_m$ of cycle type ρ,

$$\{ \text{ marked } (\rho, l)\text{-tableaux on } \mu \} \overset{1:1}{\longleftrightarrow} \left\{ \{T\}\text{: a tabloid on } \mu \,\middle|\, \sigma\{T\} = \{T\} a_{\mu,l}^{-j} \right\} .$$

How to construct 1-1.

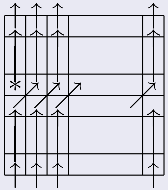
1	1	3*
1	1	3
1*	1	3
2	2	
2*	2	
2	2	

$\longmapsto$ {

3	6	13
2	5	15
1	4	14
8	11	
7	10	
9	12	

}

For each rectangle whose boxes has i,
put numbers on the boxes of μ from the marked box as follows:



Tabloids on μ

Tabloids are row-equivalent classes of numberings on μ, i.e., $\{T\} = \{T'\}$ if corresponding rows of T and T contain the same elements.

Example

$$\left\{ \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline 10 & 11 & \multicolumn{1}{c}{} \\ \cline{1-2} 12 & 13 & \multicolumn{1}{c}{} \\ \cline{1-2} 14 & 15 & \multicolumn{1}{c}{} \\ \cline{1-2} \end{array} \right\} = \left\{ \begin{array}{|c|c|c|} \hline 2 & 1 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline 10 & 11 & \multicolumn{1}{c}{} \\ \cline{1-2} 12 & 13 & \multicolumn{1}{c}{} \\ \cline{1-2} 14 & 15 & \multicolumn{1}{c}{} \\ \cline{1-2} \end{array} \right\} \neq \left\{ \begin{array}{|c|c|c|} \hline 4 & 2 & 3 \\ \hline 1 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline 10 & 11 & \multicolumn{1}{c}{} \\ \cline{1-2} 12 & 13 & \multicolumn{1}{c}{} \\ \cline{1-2} 14 & 15 & \multicolumn{1}{c}{} \\ \cline{1-2} \end{array} \right\}$$

Remark

The S_n-module M^μ whose basis is the set of tabloids on μ is isomorphic to the Springer module R_μ as S_n-modules.

Remark

$a_{\mu,l}^{-1}$ acts on tabloids from the right as follows:

$$\left\{ \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline 10 & 11 & \multicolumn{1}{c}{} \\ \cline{1-2} 12 & 13 & \multicolumn{1}{c}{} \\ \cline{1-2} 14 & 15 & \multicolumn{1}{c}{} \\ \cline{1-2} \end{array} \right\} a_{\mu,l}^{-1} = \left\{ \begin{array}{|c|c|c|} \hline 7 & 8 & 9 \\ \hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 14 & 15 & \multicolumn{1}{c}{} \\ \cline{1-2} 10 & 11 & \multicolumn{1}{c}{} \\ \cline{1-2} 12 & 13 & \multicolumn{1}{c}{} \\ \cline{1-2} \end{array} \right\}$$

Remark

For $\sigma_\rho = (1,2,3,4,5,6)(7,8,9,10,11,12)(13,14,15)$

$$\left\{ \begin{array}{|c|c|c|} \hline 3 & 6 & 13 \\ \hline 2 & 5 & 15 \\ \hline 1 & 4 & 14 \\ \hline 8 & 11 & \\ \cline{1-2} 7 & 10 & \\ \cline{1-2} 9 & 12 & \\ \cline{1-2} \end{array} \right\} a_{\mu,l}^{-1} = \left\{ \begin{array}{|c|c|c|} \hline 1 & 4 & 14 \\ \hline 3 & 6 & 13 \\ \hline 2 & 5 & 15 \\ \hline 9 & 12 & \\ \cline{1-2} 8 & 11 & \\ \cline{1-2} 7 & 10 & \\ \cline{1-2} \end{array} \right\}$$

$$\sigma \left\{ \begin{array}{|c|c|c|} \hline 3 & 6 & 13 \\ \hline 2 & 5 & 15 \\ \hline 1 & 4 & 14 \\ \hline 8 & 11 & \\ \cline{1-2} 7 & 10 & \\ \cline{1-2} 9 & 12 & \\ \cline{1-2} \end{array} \right\} = \left\{ \begin{array}{|c|c|c|} \hline 4 & 1 & 14 \\ \hline 3 & 6 & 13 \\ \hline 2 & 5 & 15 \\ \hline 9 & 12 & \\ \cline{1-2} 8 & 11 & \\ \cline{1-2} 10 & 7 & \\ \cline{1-2} \end{array} \right\}$$