Combinatorial representation theory of algebras: the example of j-trivial monoids

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Abstract. The representation theory of algebras is a very important source of interesting combinatorics. The symmetric groups leading to the combinatorics of tableaux is certainly the most striking example, but it is far from being unique: other examples include the various Hecke algebras, descent algebras... Another interesting feature of the representation theory of the finite dimensional algebras is that it is mostly effective. As a consequence, with the appropriate tools one can very easily use computers for exploration.

The goal of the talk is to discuss these features together with the simple remark that several recently studied algebras are in fact monoid algebras: examples are 0-Hecke algebras, degenerated Ariki-Koiki algebras, Solomon-Tits algebras. Apparently, the fact that they are indeed monoid algebras wasn't used in those studies. However, from recent results in semigroups theory it seem that a lot of representation theory of a semigroup algebra is of combinatorial nature (provided the representation theory of groups is known). One can expect, for example, that for aperiodic semigroup (semigroup which doesn't contains non trivial groups), most of the combinatorial information (dimensions of the simple/projective indecomposable, induction/restriction constants/Cartan's invariants) can be computed without using any linear algebra.

In this talk, we will focus on the so-called J-trivial monoids, which are the monoids M such that the product has the following triangular properties: there exists a partial ordering $\leq$ on M such that for a $x, y \in M$, one had $xy \leq x$ and $xy \leq y$. A typical example is the 0-Hecke monoid of a Coxeter group. We will show that for such a monoid, most of the combinatorial data of the representation theory including the Cartan's invariant matrix and the quiver can be expressed by counting particular kinds of elements in the monoid itself.

This is a joint work with Tom Denton, Anne Schilling and Nicolas Thiéry.