Recent developments on log-concavity and q-log-concavity of combinatorial polynomials

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Abstract. In this talk, I wish to report some recent work with my students and colleagues at Nankai University on log-concavity and q-log-concavity of combinatorial polynomials. While this is a classical subject of algebraic combinatorics, interesting problems and techniques continue to emerge.
(1) We proved the unimodality conjecture on balanced colorings of the n-cube proposed by Palmer, Read and Robinson, and obtained a log-concavity theorem for sufficiently large n.
(2) We proved the ratio monotone property of the Boros-Moll polynomials which is stronger than the log-concavity. We further proved the 2-log-concavity which was considered as a difficult problem. The 2-log-convexity of the Apery numbers has been established. We obtained the reverse ultra log-concavity of the Boros-Moll polynomials, and confirmed Moll's minimum conjecture. A combinatorial approach has been found to justify the log-concavity and other properties of the Boros-Moll polynomials.
(3) By using the Littlewood-Richardson rule, we obtained certain Schur positivity results that lead to a proof of the q-log-convexity conjecture for the Narayana polynomials.
(4) By establishing the strong q-log-concavity of q-Narayana numbers $N_q(n,k)$ for fixed k, we confirmed the 2-fold case of the q-log-concavity conjecture for the Gaussian coefficients proposed by McNamara and Sagan.
(5) We found a unified approach to the q-log-convexity of the Bell polynomials, the Bessel polynomials, the Ramanujan polynomials and the Dowling polynomials.
(6) We shall also mention some open problems.