

# *The Worm order and its applications*

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**Abstract.** Let  $x$  and  $y$  be two words in a linearly-ordered alphabet (such as the real numbers). We say that  $x$  is below  $y$  in the worm order if they can be “scheduled” in such a way that  $x$  is always less than or equal to  $y$ . It turns out that in any submodular system there is a maximal chain that is minimal in the worm order, among all paths from 0 to 1. One consequence is a set of general conditions under which parallel scheduling can be done without backward steps. Among the applications are a fast algorithm for scheduling multiple processes without overusing a resource; a theorem about searching for a lost child in a forest; and a closed-form expression for the probability of escaping from the origin in a form of coordinate percolation. Joint work in part with Graham Brightwell (LSE) and in part with Lizz Moseman (USMA).