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Schur and Macdonald processes

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The Schur processes are measures on sequences of partitions whose weights are defined as products of suitable skew Schur functions. They were introduced in [1] for the purpose of asymptotic analysis of large random plane partitions. The Schur processes are indispensable in asymptotic analysis of a variety of combinatorial and probabilistic models such as finding the length of the longest increasing subsequence of random permutations, domino tilings of Aztec diamonds, random growth processes in $(1+1)$ and $(2+1)$ dimensions, etc.

The Macdonald processes came into play much more recently [2]. They generalize the Schur processes in a natural way (as the Macdonald polynomials generalize the Schur ones), but their usefulness for probabilistic purposes is much less obvious.

One goal of the talk is to explain how basic facts from the theory of symmetric functions, such as Cauchy type identities and branching rules, give rise to Markov chains that preserve the class of Macdonald processes. Special cases of such Markov chains include well-known interacting particle systems called exclusion processes as well as stochastic dynamics on random stepped surfaces (equivalently, lozenge tilings).

The second goal of the talk is to show how Macdonald difference operators can be used to evaluate averages of a broad class of observables on Macdonald processes.

The combination of the construction of the Markov chains with using such observables recently lead to a significant progress in asymptotic analysis of $(1+1)$-dimensional directed polymers in random media [2, 3, 4].

References

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