From alternating sign matrices to Painlevé VI

Hjalmar Rosengren

In 1996, Kuperberg used the six-vertex model to give a simple proof of the alternating sign matrix theorem, first proved by Zeilberger. Replacing the six-vertex model by the more general eight-vertex-solid-on-solid model (or Andrews–Baxter–Forrester model), Kuperberg's approach leads to new results on three-colourings of the square lattice. The generating function for the three colours (on a square with appropriate boundary conditions) can be expressed in terms of specialized affine Lie algebra characters, which turn out to be tau functions for the Painlevé VI equation. We will describe these connections and give some concrete combinatorial consequences.

The talk will be based on the papers

H. Rosengren, *An Izergin–Korepin-type identity for the* 8VSOS *model, with applications to alternating sign matrices*, Adv. Appl. Math. 43 (2009), 137–155, `arXiv:0801.1229`,

H. Rosengren, *The three-colour model with domain wall boundary conditions*, Adv. Appl. Math. 46 (2011), 481–535, `arXiv:0911.0561`,

and on our unpublished work.

Closely related recent work of other authors include

V. V. Mangazeev and V. V. Bazhanov, *Eight-vertex model and Painlevé VI equation. II. Eigenvector results*, J. Phys. A 43 (2010) 085206, `arXiv:0912.2163`,

A. V. Razumov and Yu. G. Stroganov, *A possible combinatorial point for* XYZ*-spin chain*, Theor. Math. Phys. 164 (2010), 977–991, `arXiv:0911.5030`,

P. Zinn-Justin, *Sum rule for the eight-vertex model on its combinatorial line*, `arXiv:1202.4420`.