Promotion and Rowmotion

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Rowmotion

What's in a name?

1. F (Brouwer and Schrijver)

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2. f (Deza and Fukuda, Fon-Der-Flaass and Cameron)

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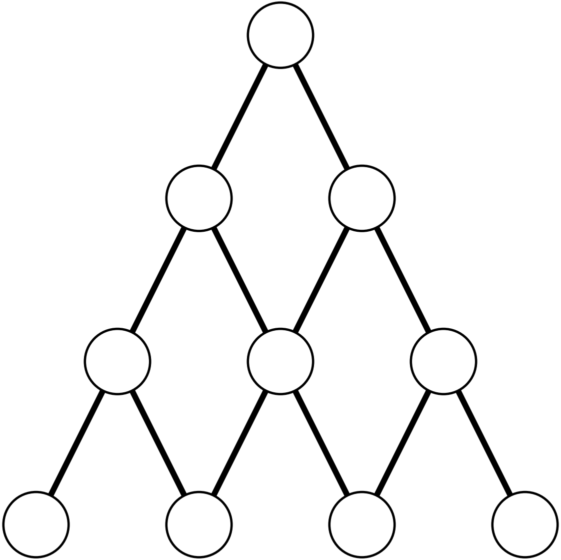
5. Panyushev action (Bessis and Reiner)
6. Panyushev complement (Armstrong, Stump, and Thomas)
7. Fon-Der-Flaass action (Rush and Shi)

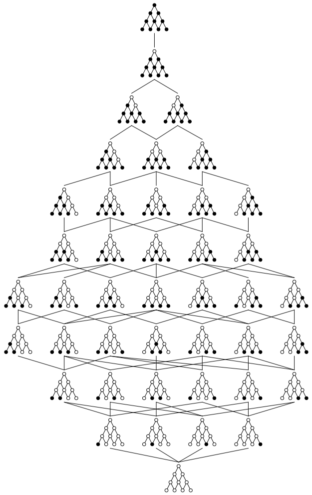
What's in a name?

"What's in a name? That which we call **rowmotion**
By any other name would smell as sweet."

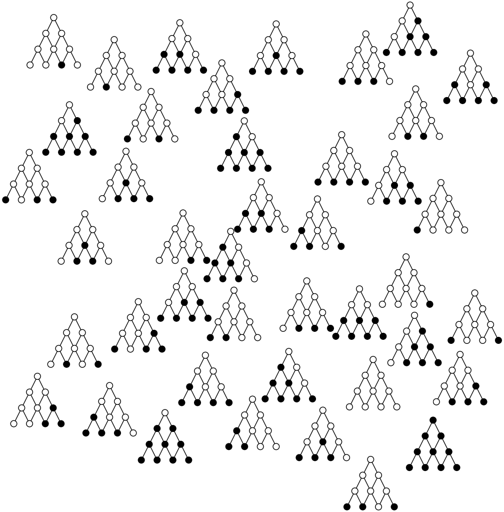
A poset P



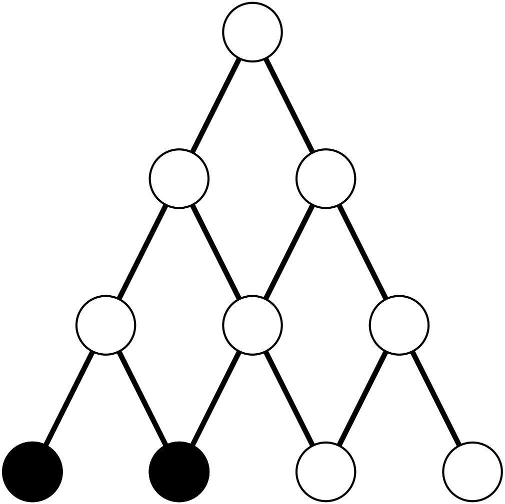
The distributive lattice of order ideals $J(P)$



The set $J(P)$

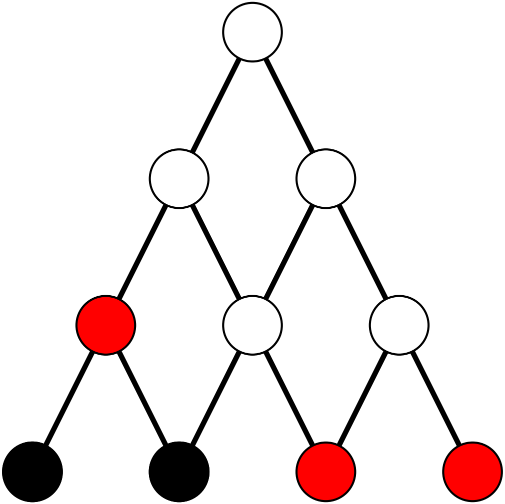


Computing Rowmotion



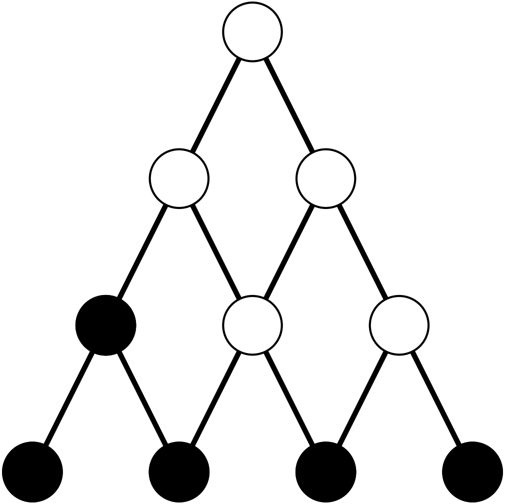
An order ideal I

Computing Rowmotion



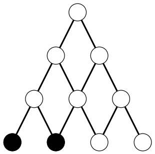
Find the **minimal** elements of P not in I

Computing Rowmotion



And use them to generate a new order ideal **Row(I)**

An Orbit under Rowmotion



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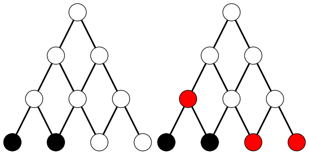

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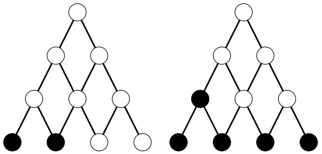

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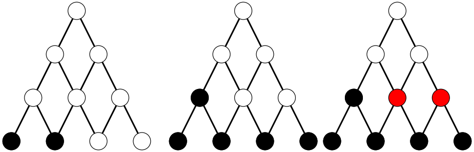

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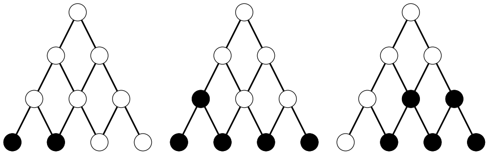

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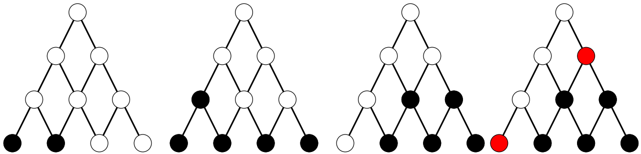

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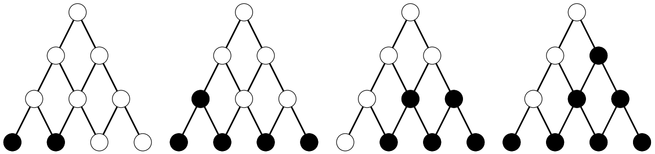

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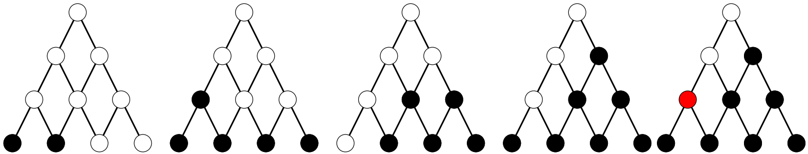

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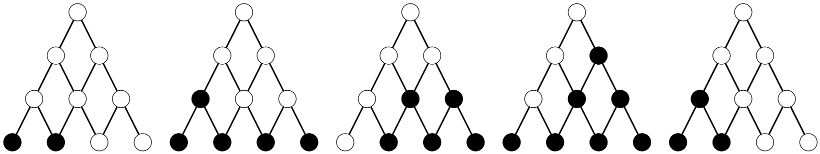

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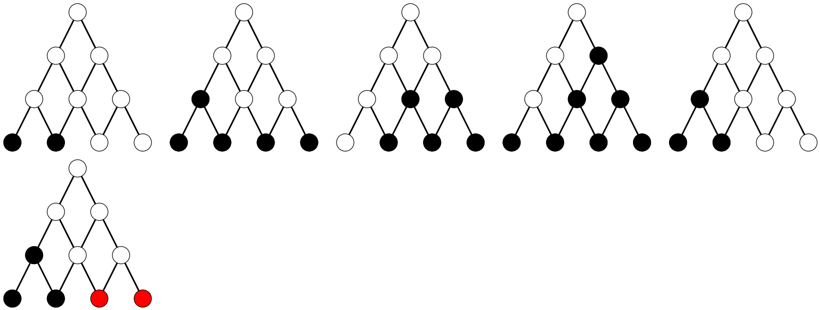

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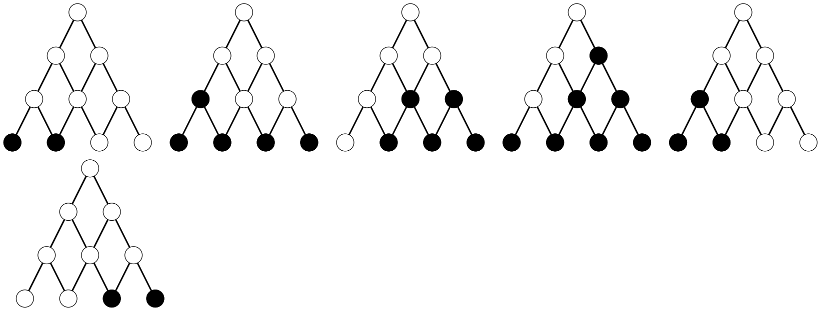

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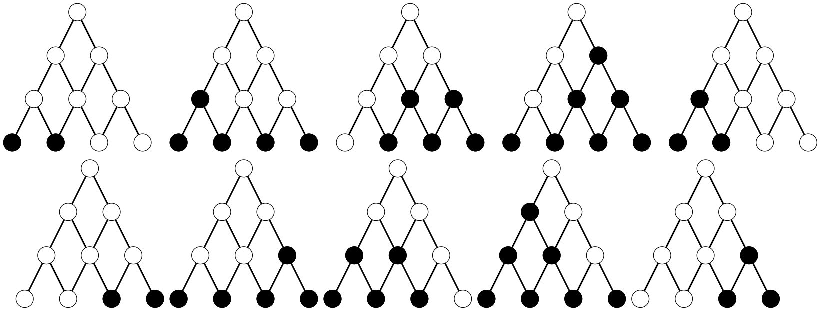
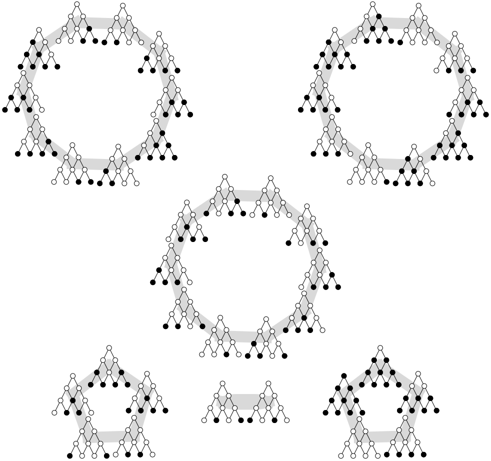


Rowmotion Computed



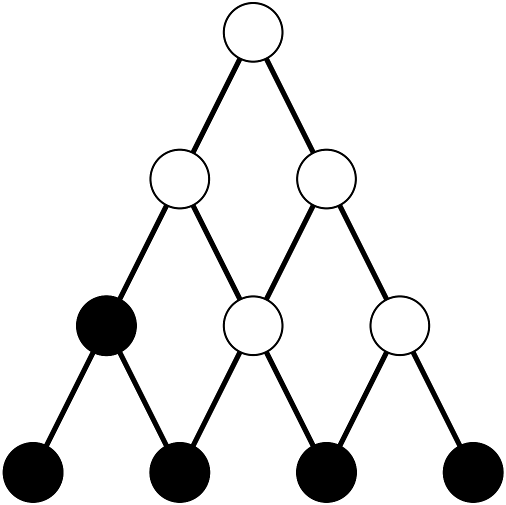
The orbits of $J(P)$ under rowmotion.

Philosophy

Combinatorial objects with "well-behaved" cyclic actions should have models where the cyclic action becomes rotation.

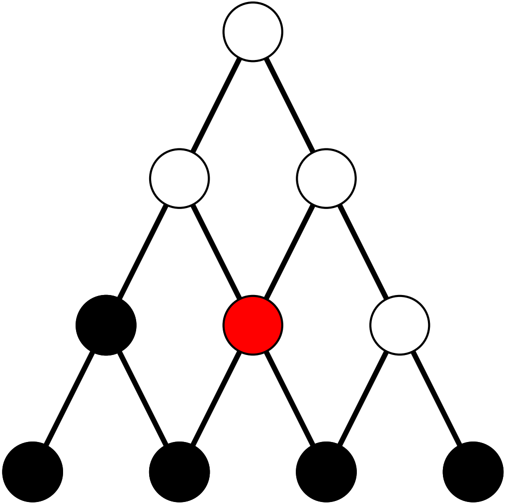
The Toggle Group

Toggles



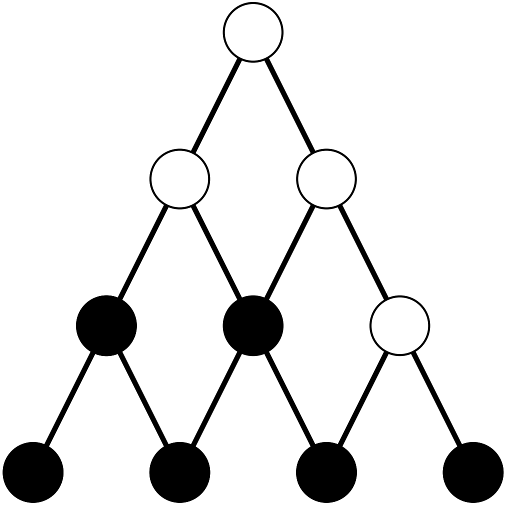
Toggles t_p, with $p \in P$.

Toggles



Toggles t_p add p when possible.

Toggles



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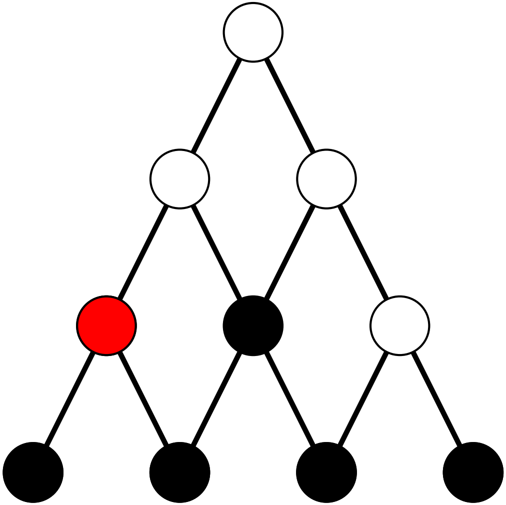

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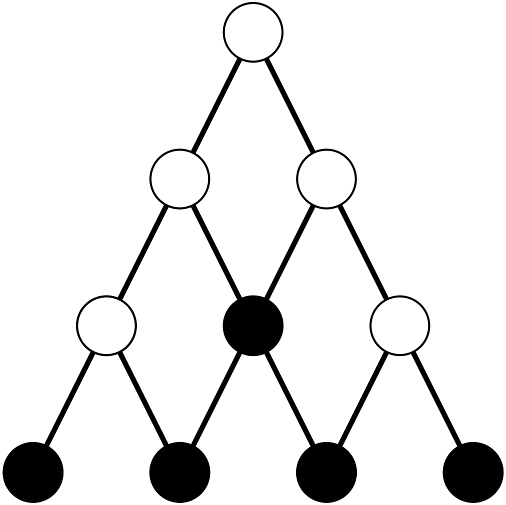

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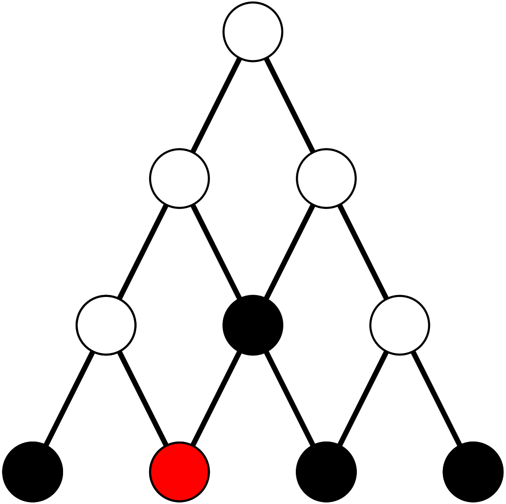
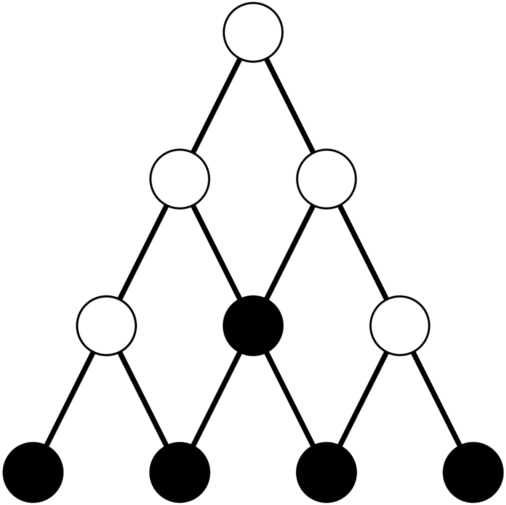


Toggles t_p do nothing otherwise.

Toggles



Toggles t_p do nothing otherwise.

Definition (P. Cameron and D. Fon-der-Flaass)

The **toggle group** $T(\mathcal{P})$ of a poset $\mathcal{P}$ is the subgroup of the permutation group $\mathfrak{S}_{J(P)}$ generated by $\{t_p\}_{p\in\mathcal{P}}$.

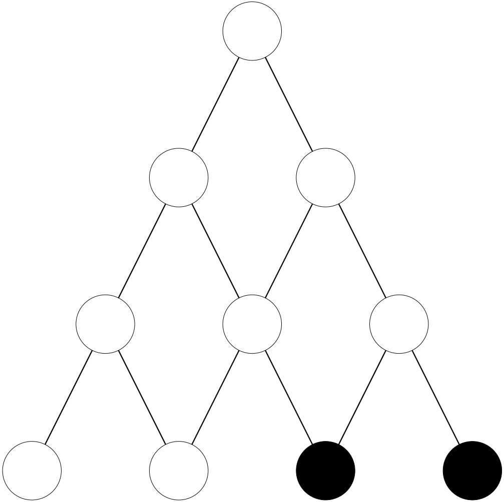
More Philosophy

Philosophy

If we have combinatorial objects encoded as order ideals of some poset, we can model known actions using elements in the toggle group.

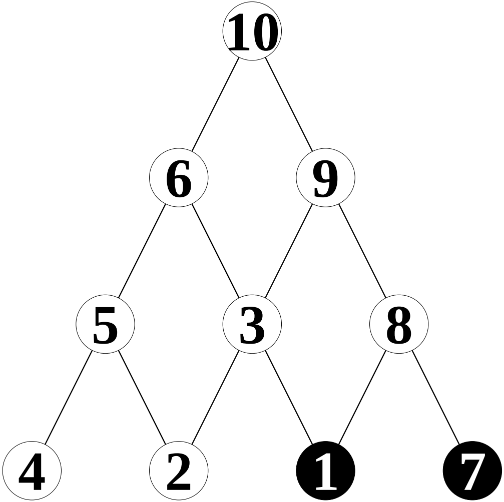
Computing (Inverse) Rowmotion

An order ideal I

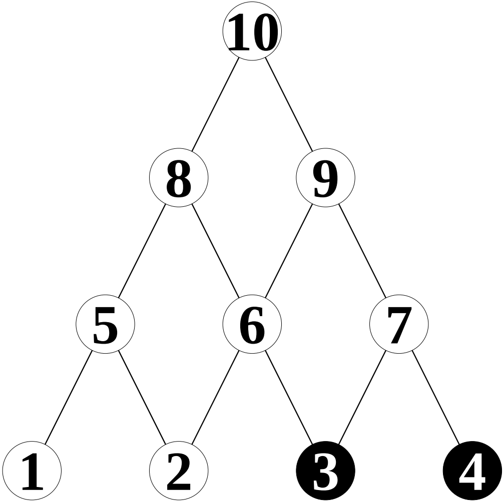


Computing (Inverse) Rowmotion

Fix a **linear extension** of P

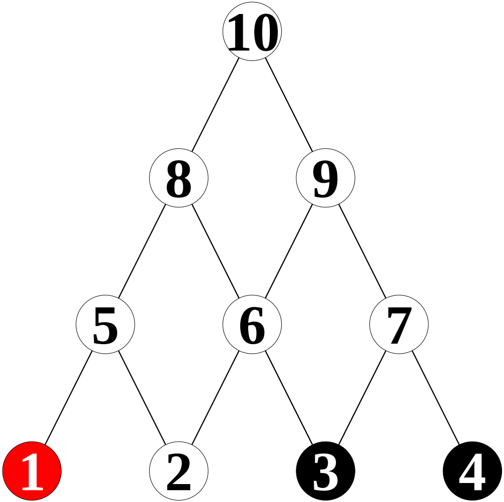


Fix **this** linear extension of P



Computing (Inverse) Rowmotion

Toggle in order to get $\mathbf{Row}^{-1}(\mathbf{I})$



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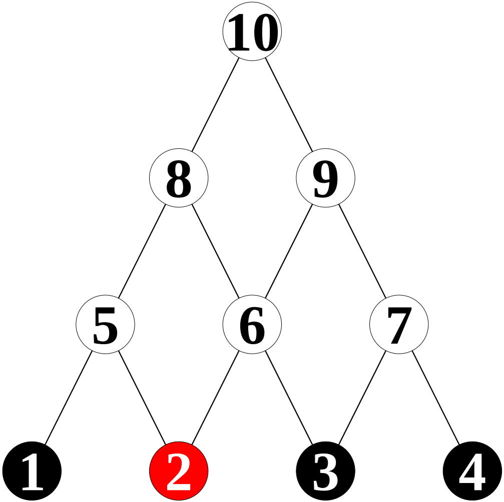

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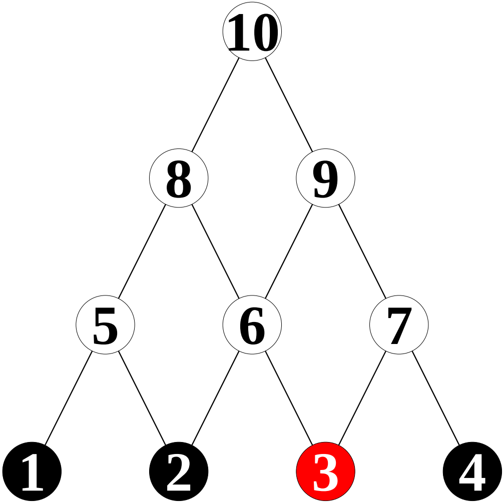

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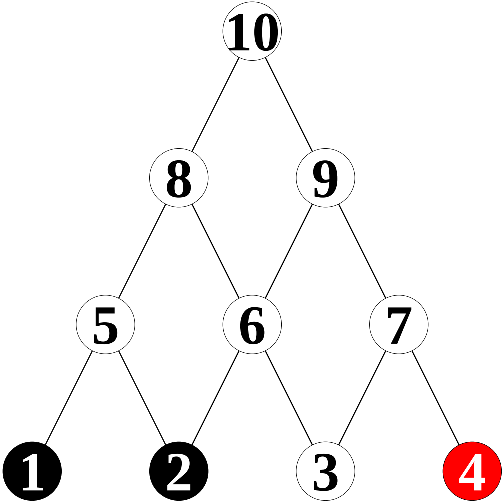
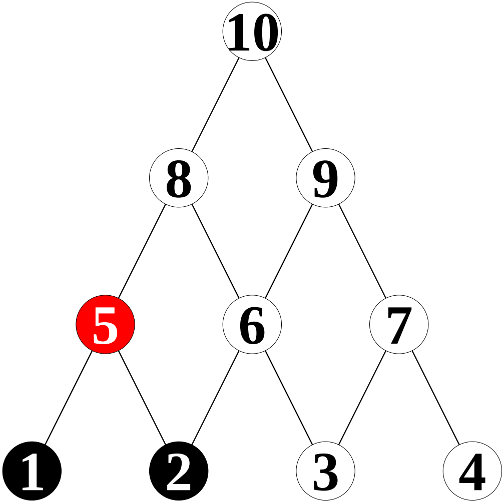


Computing (Inverse) Rowmotion

Toggle in order to get **Row**$^{-1}$(**I**)



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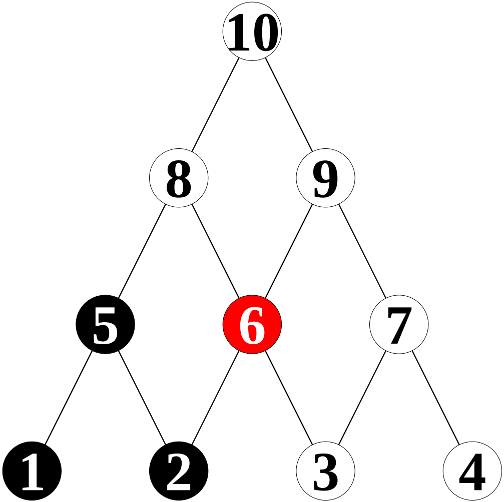

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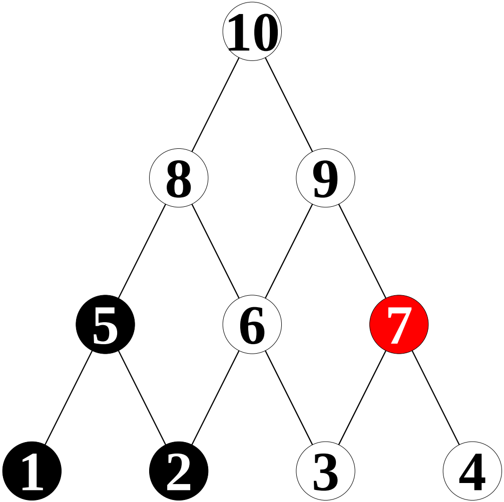

Computing (Inverse) Rowmotion

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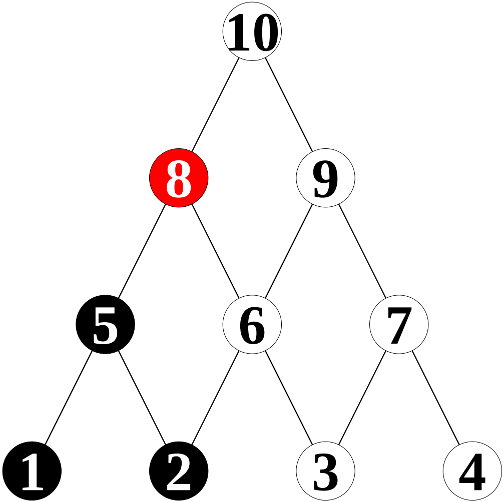

Computing (Inverse) Rowmotion

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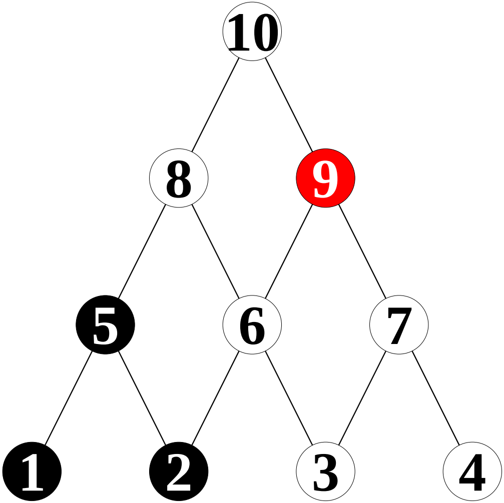

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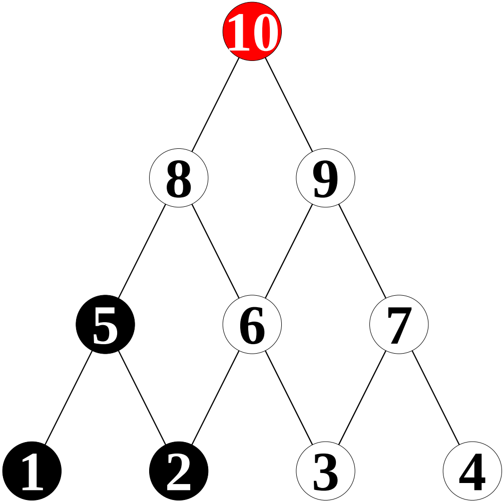

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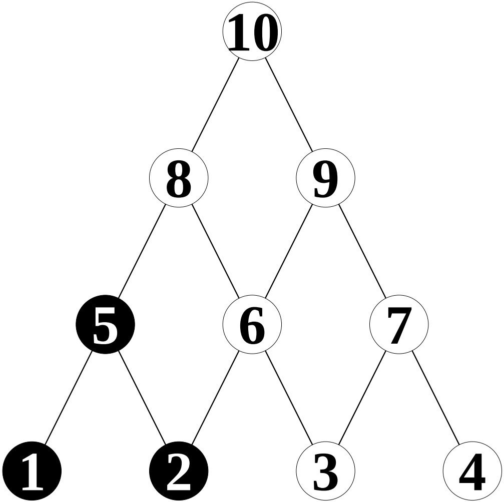

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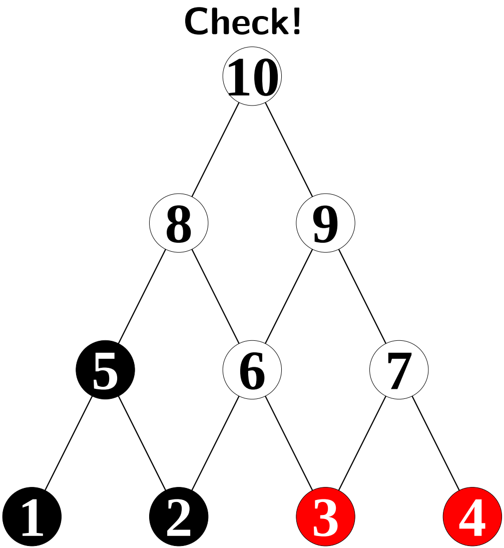
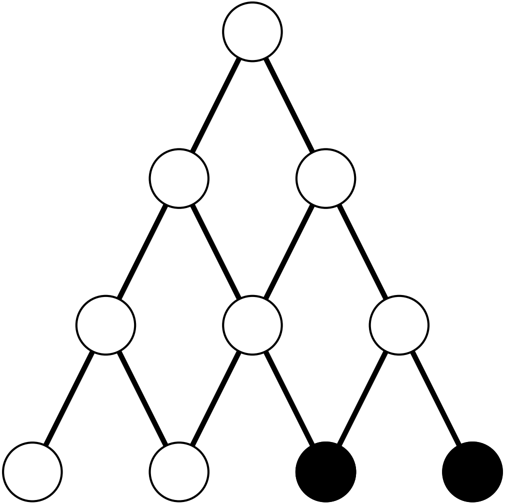


Computing (Inverse) Rowmotion

Toggle by **rows** to get $\mathbf{Row}^{-1}(\mathbf{I})$



Computing (Inverse) Rowmotion

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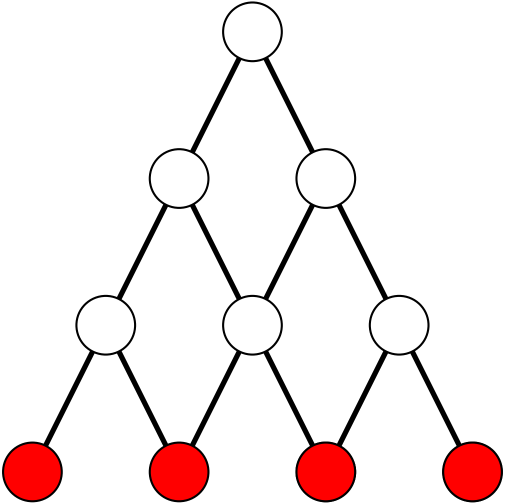

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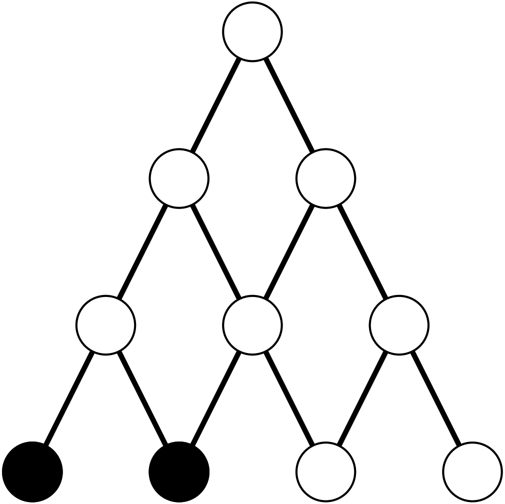

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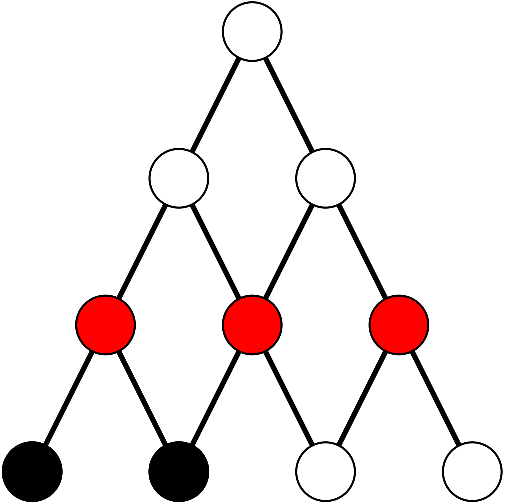

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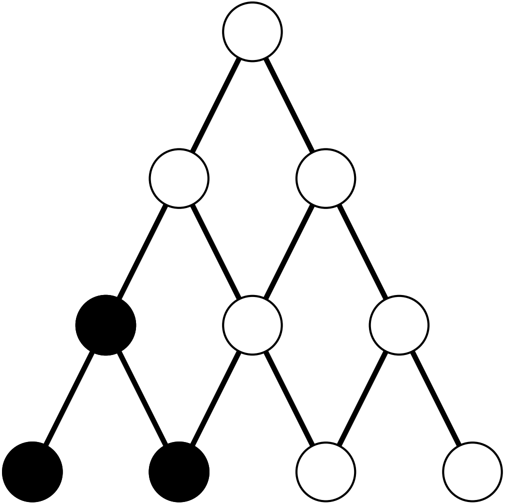

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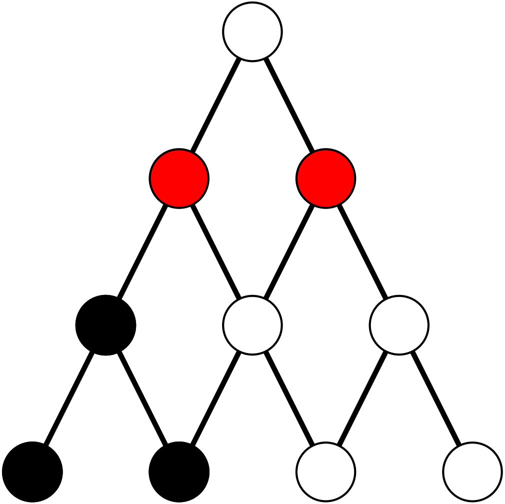

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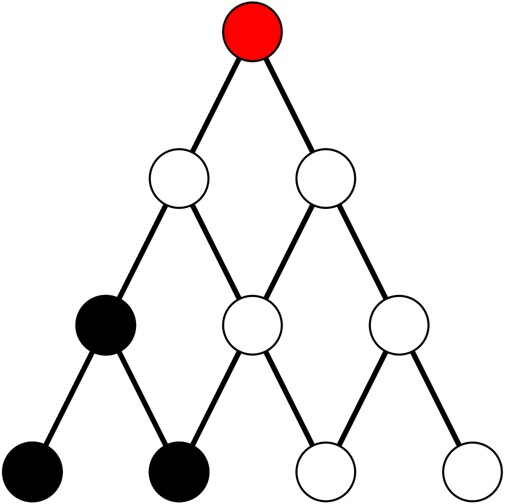

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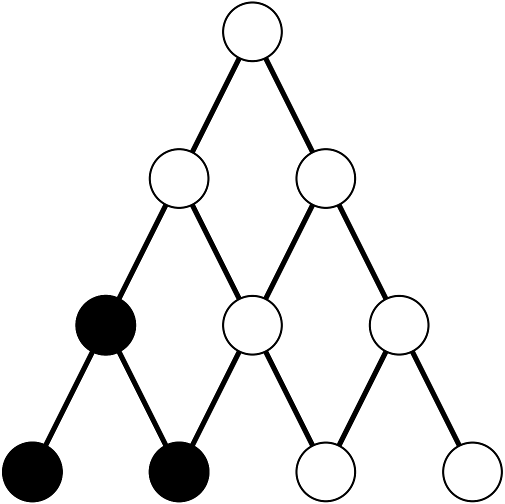


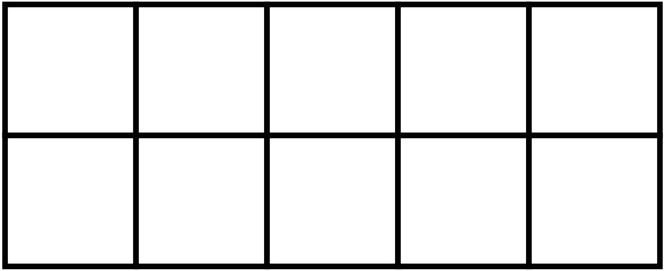
Strategy

Strategy

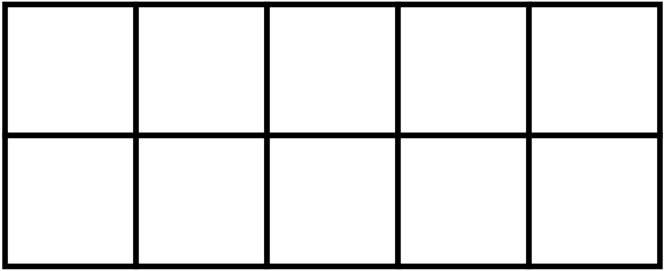
Find a "good" conjugate to rowmotion in the toggle group.

Promotion

SYT



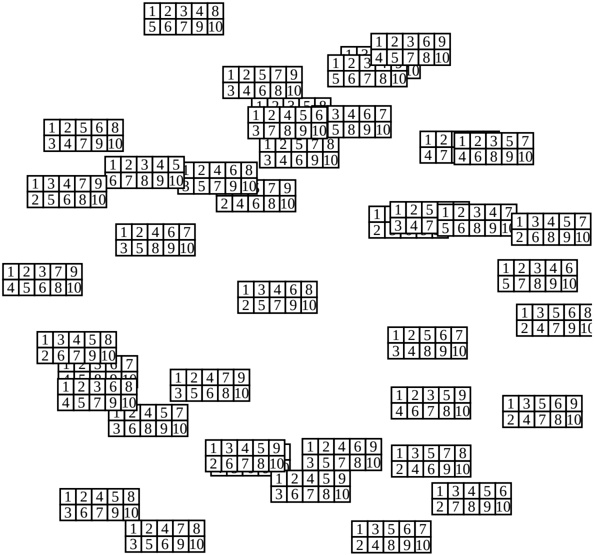
SYT



Question

How many linear extensions are there of this shape?

Catalan Many!



Promotion is $\prod_i \rho_i$, where
ρ_i swaps i and $i+1$ **when possible**.

1	2	3	4	7
5	6	8	9	10

Computing Promotion

Promotion is $\prod_i \rho_i$, where
ρ_1 swaps 1 and 2 when possible.

1	2	3	4	7
5	6	8	9	10

Computing Promotion

Promotion is $\prod_i \rho_i$, where
ρ_2 swaps 2 and 3 when possible.

1	2	3	4	7
5	6	8	9	10

Computing Promotion

Promotion is $\prod_i \rho_i$, where
ρ_3 swaps 3 and 4 when possible.

1	2	3	4	7
5	6	8	9	10

Promotion is $\prod_i \rho_i$, where
ρ_4 swaps 4 and 5 when possible.

1	2	3	4	7
5	6	8	9	10

Promotion is $\prod_i \rho_i$, where
ρ_4 swaps 4 and 5 **when possible**.

1	2	3	5	7
4	6	8	9	10

Computing Promotion

Promotion is $\prod_i \rho_i$, where
ρ_5 swaps 5 and 6 when possible.

1	2	3	5	7
4	6	8	9	10

Computing Promotion

Promotion is $\prod_i \rho_i$, where
ρ_5 swaps 5 and 6 when possible.

1	2	3	6	7
4	5	8	9	10

Computing Promotion

Promotion is $\prod_i \rho_i$, where
ρ_6 swaps 6 and 7 when possible.

1	2	3	6	7
4	5	8	9	10

Promotion is $\prod_i \rho_i$, where
ρ_7 swaps 7 and 8 when possible.

1	2	3	6	7
4	5	8	9	10

Computing Promotion

Promotion is $\prod_i \rho_i$, where
ρ_7 swaps 7 and 8 when possible.

1	2	3	6	8
4	5	7	9	10

Computing Promotion

Promotion is $\prod_i \rho_i$, where
ρ_8 swaps 8 and 9 when possible.

1	2	3	6	8
4	5	7	9	10

Computing Promotion

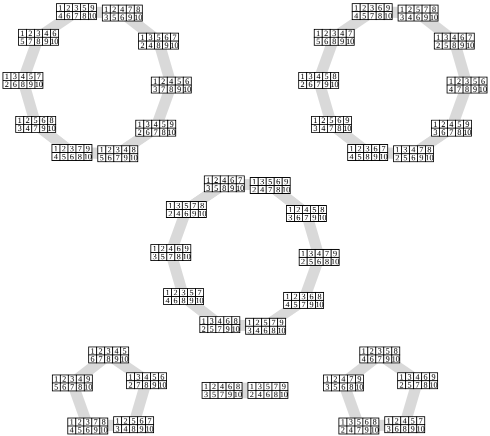
Promotion is $\prod_i \rho_i$, where
ρ_9 swaps 9 and 10 when possible.

1	2	3	6	9
4	5	7	8	10

Promotion is $\prod_i \rho_i$.

1	2	3	6	9
4	5	7	8	10

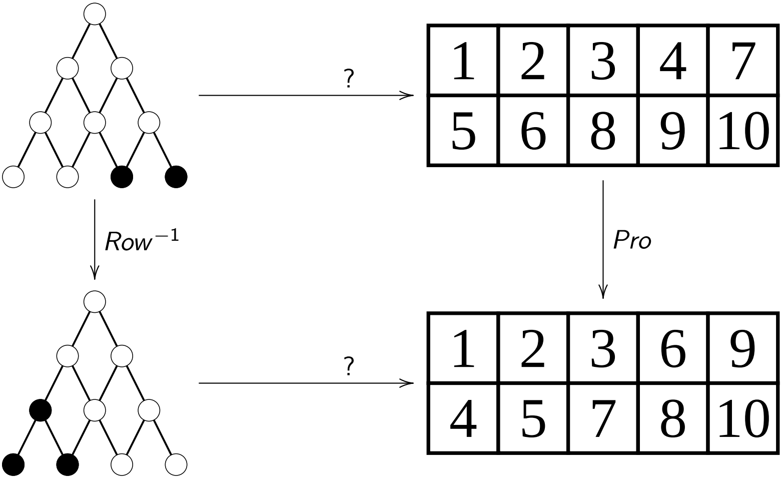
Promotion Computed



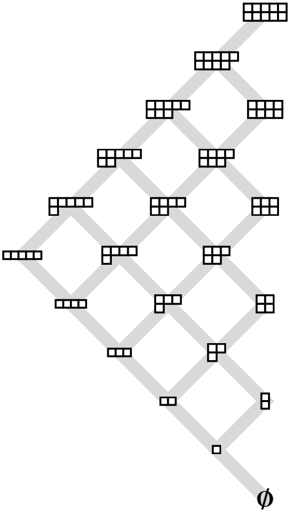
The orbits of SYT of shape (5,5) under promotion.

Promotion and Rowmotion

Promotion and Rowmotion



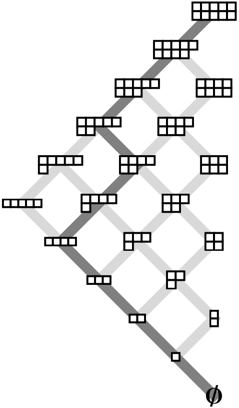
2-Rowed Ferrers Diagrams

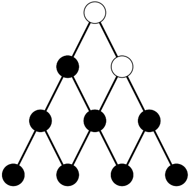


Promotion in the Toggle Group

SYT define **paths** which trace out **order ideals**

1	2	3	4	7
5	6	8	9	10

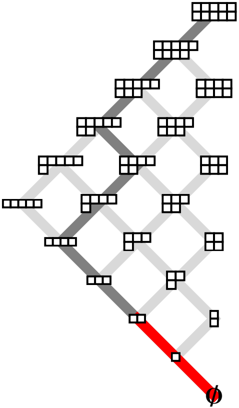


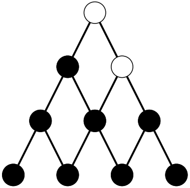


Promotion in the Toggle Group

Toggle the order ideals by **columns** to get **Pro(I)**

1	2	3	4	7
5	6	8	9	10





Promotion in the Toggle Group

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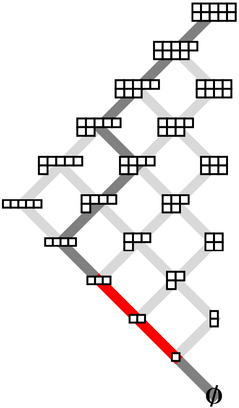

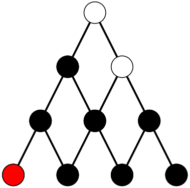

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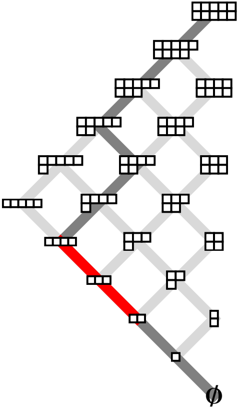

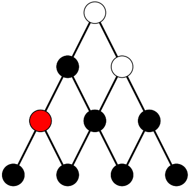

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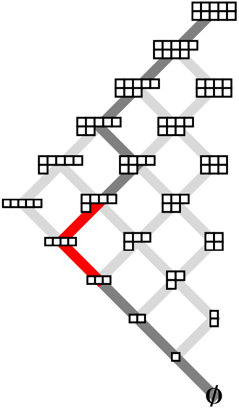

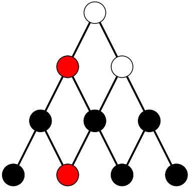

Promotion in the Toggle Group

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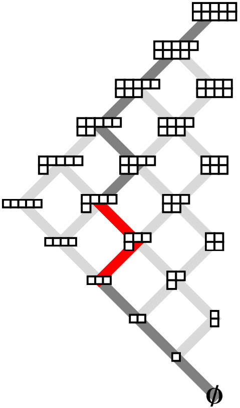

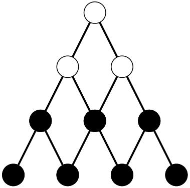

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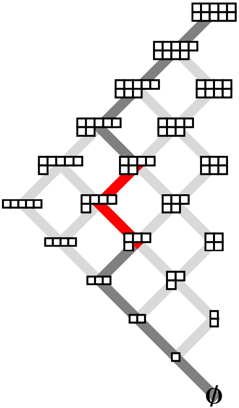

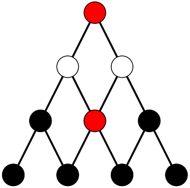

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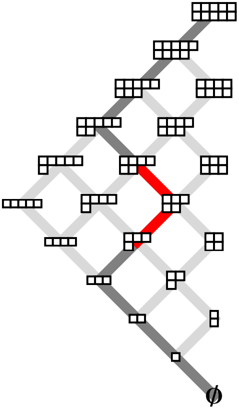

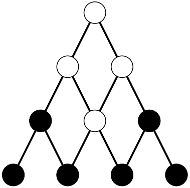

Promotion in the Toggle Group

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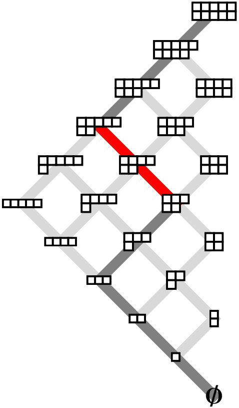

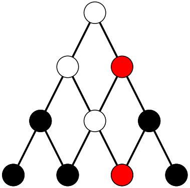

Promotion in the Toggle Group

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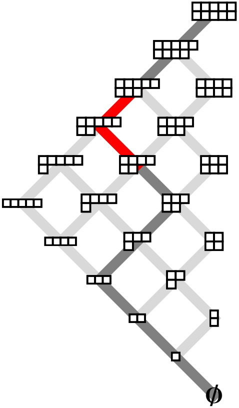

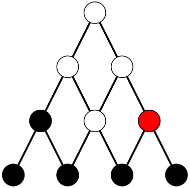

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Promotion in the Toggle Group

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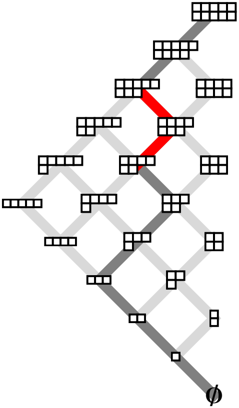

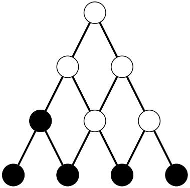

Promotion in the Toggle Group

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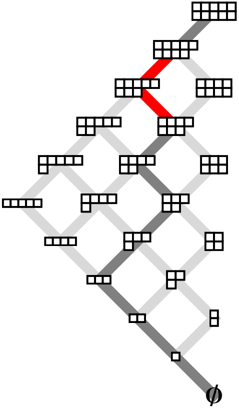

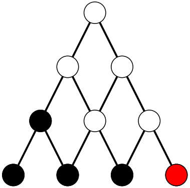

Promotion in the Toggle Group

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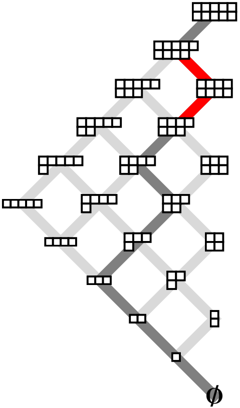
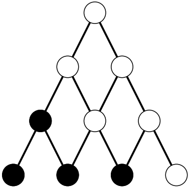
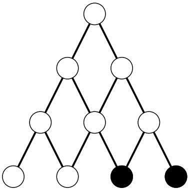




Promotion and Rowmotion



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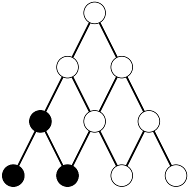
5	6	8	9	10

Row^{-1}

Pro



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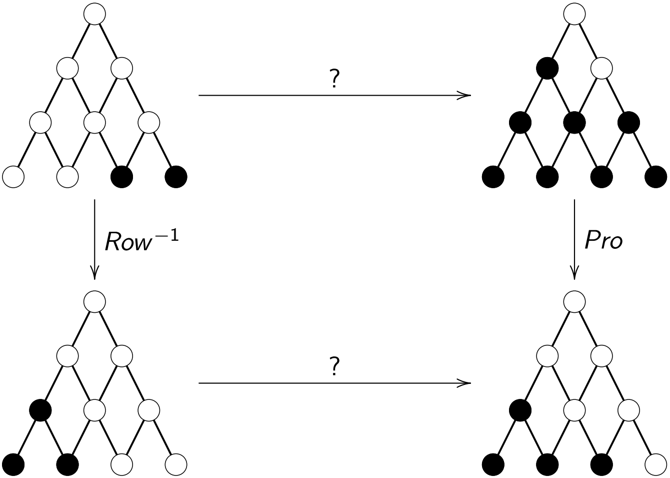
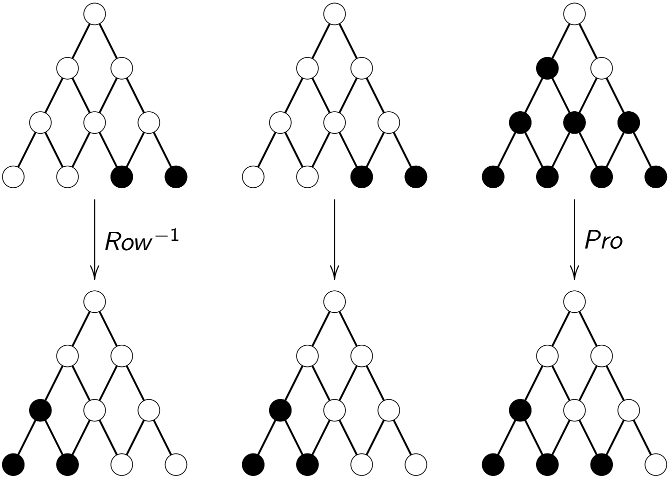
1	2	3	6	9
4	5	7	8	10

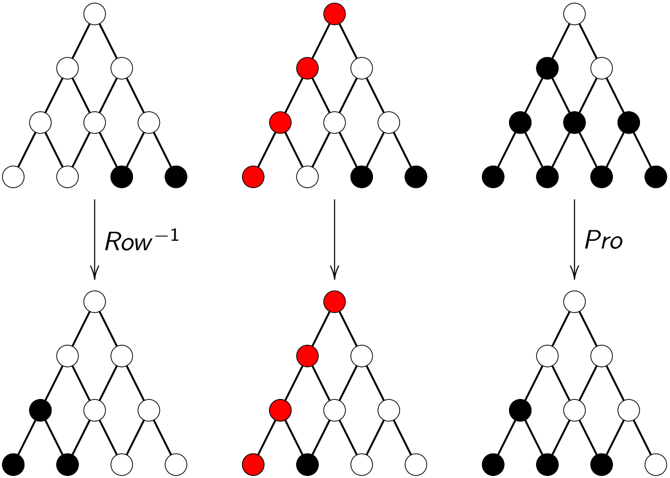
Promotion and Rowmotion



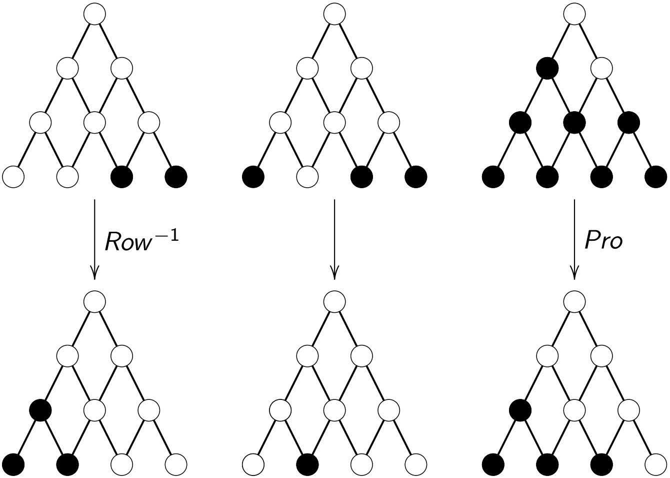
Promotion and Rowmotion



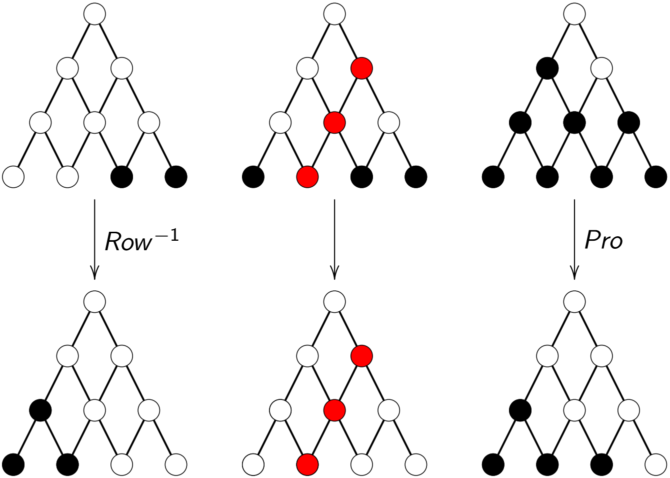
Promotion and Rowmotion



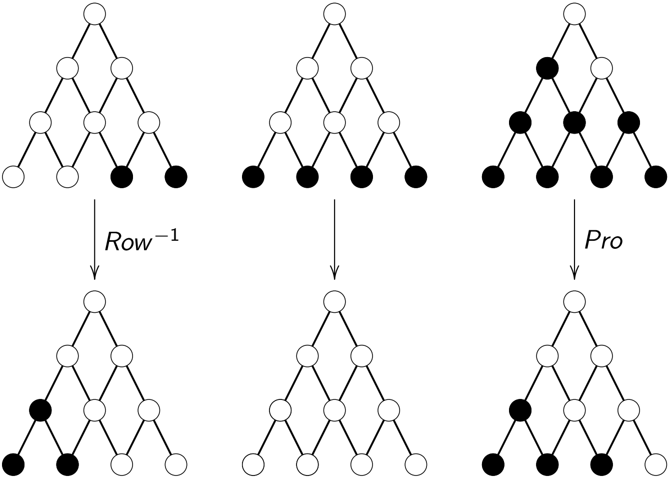
Promotion and Rowmotion



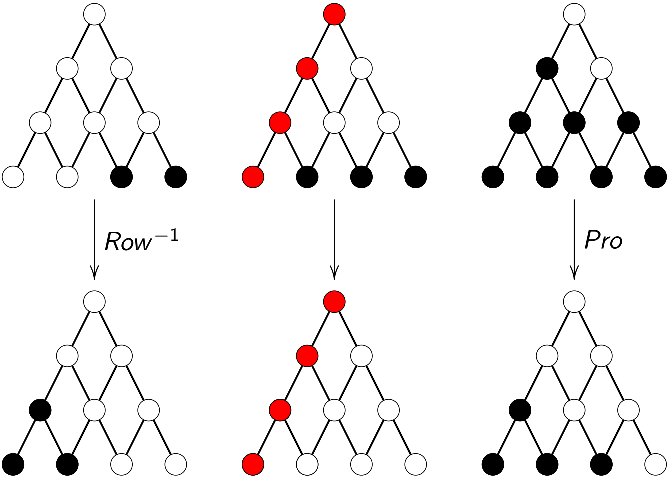
Promotion and Rowmotion



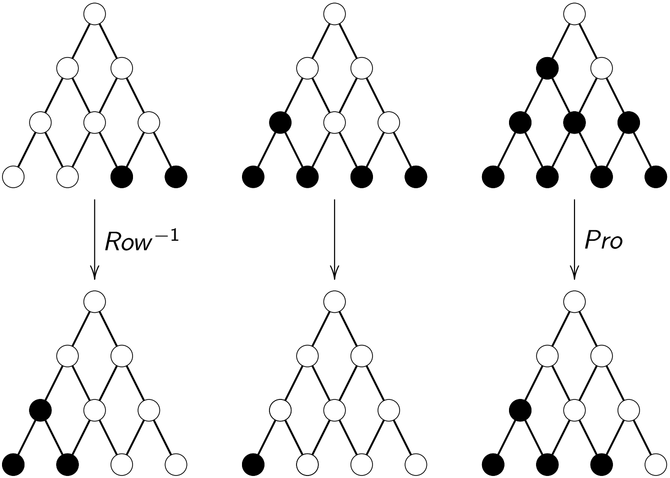
Promotion and Rowmotion



Promotion and Rowmotion



Promotion and Rowmotion



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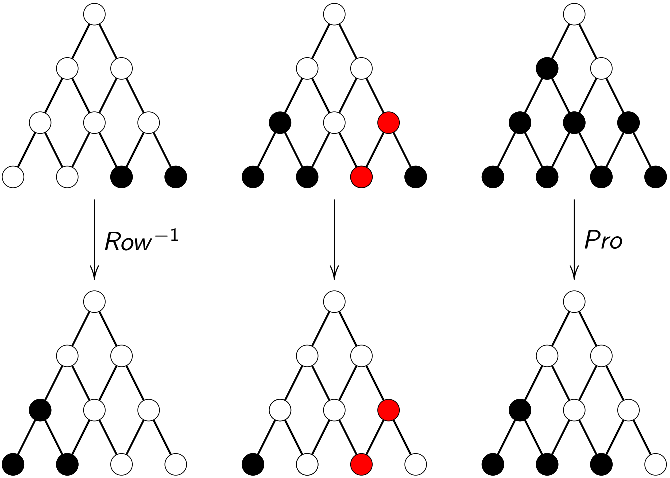
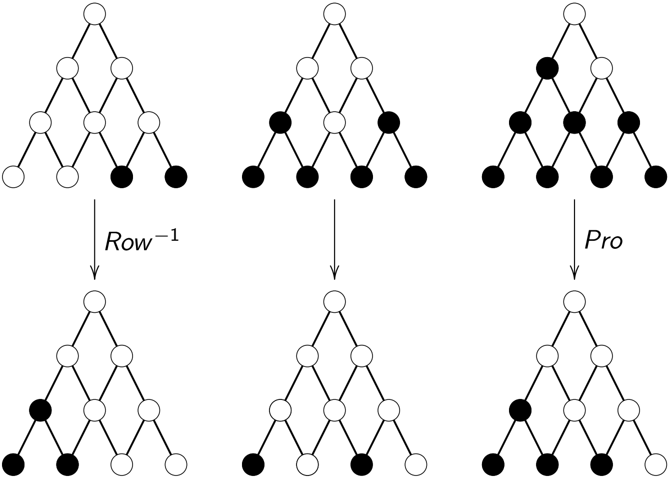


Promotion and Rowmotion



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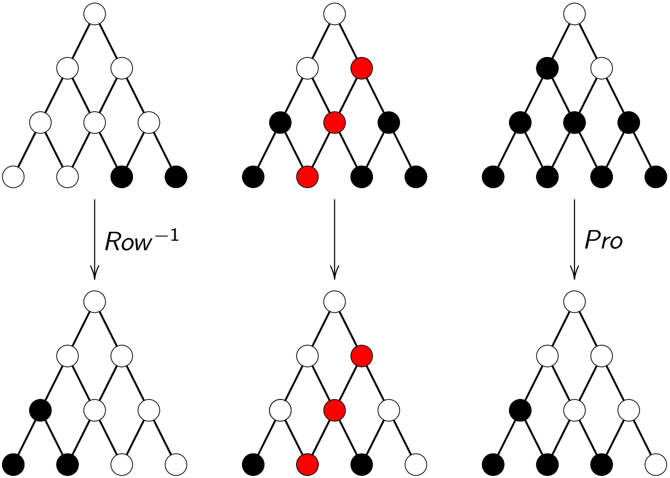
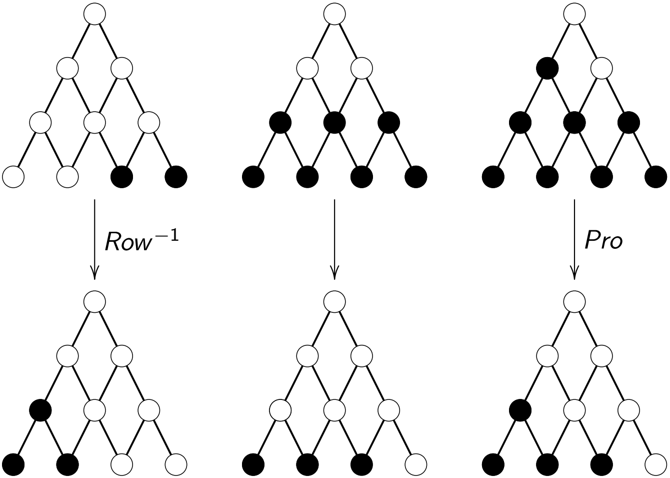
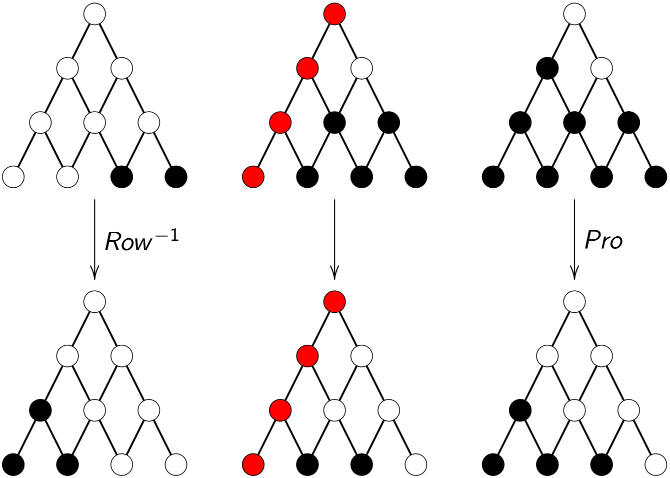


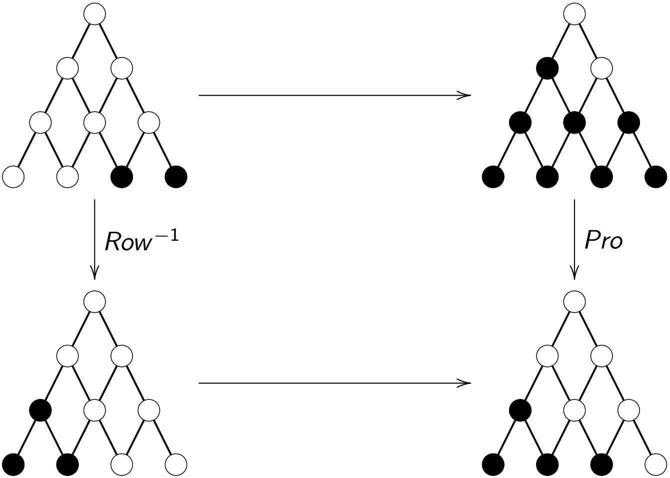
Promotion and Rowmotion



Promotion and Rowmotion



Promotion and Rowmotion



Some of our favorite objects are order ideals

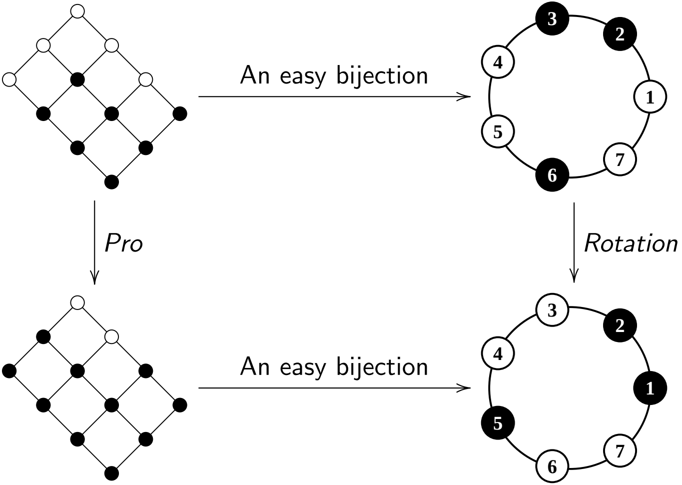
Or: Some results

Philosophy

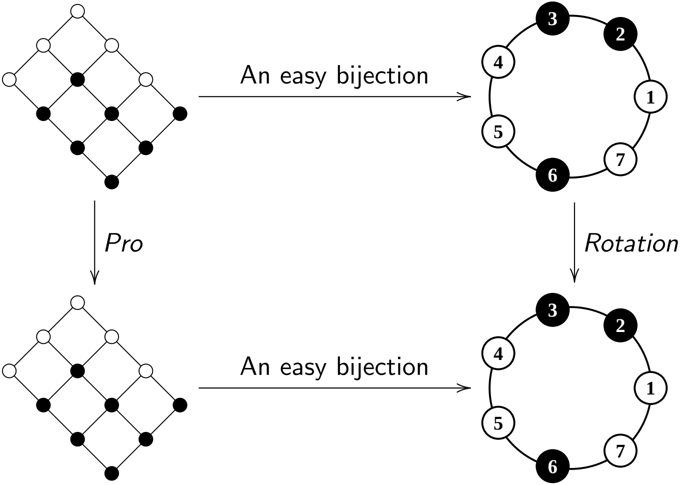
Philosophy

1. Combinatorial objects with "well-behaved" cyclic actions should have models where the cyclic action becomes rotation.
2. If we have combinatorial objects encoded as order ideals of some poset, we can model known actions using elements in the toggle group.

Binomial



Binomial


An easy bijection
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Pro
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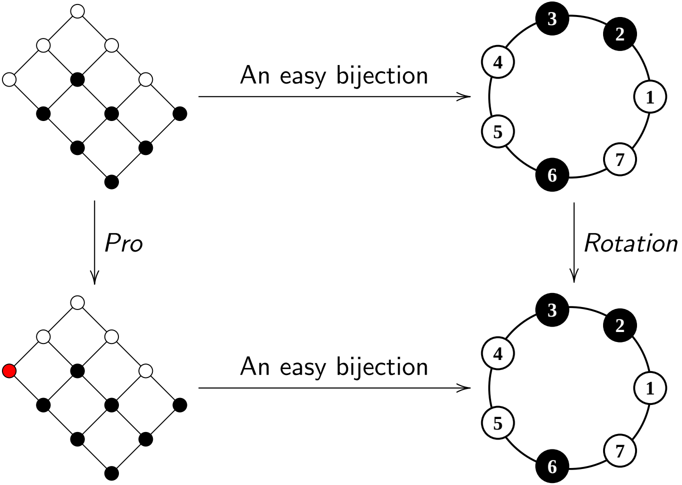
An easy bijection

Binomial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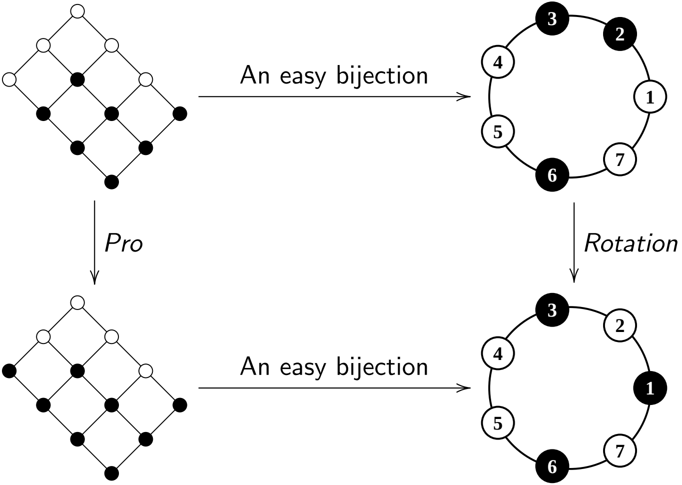

An easy bijection
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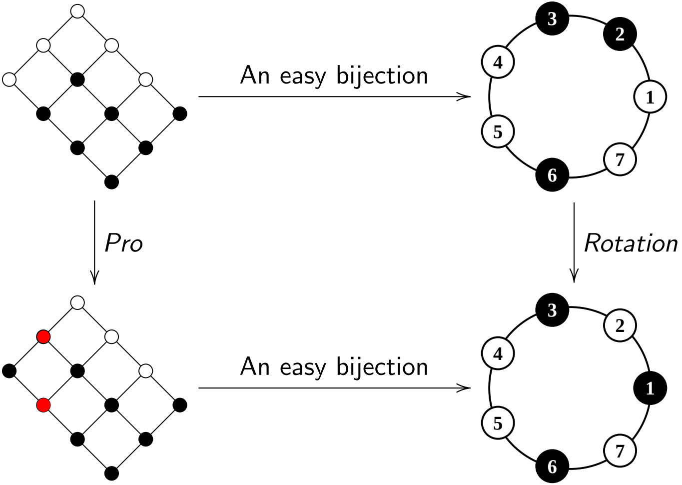
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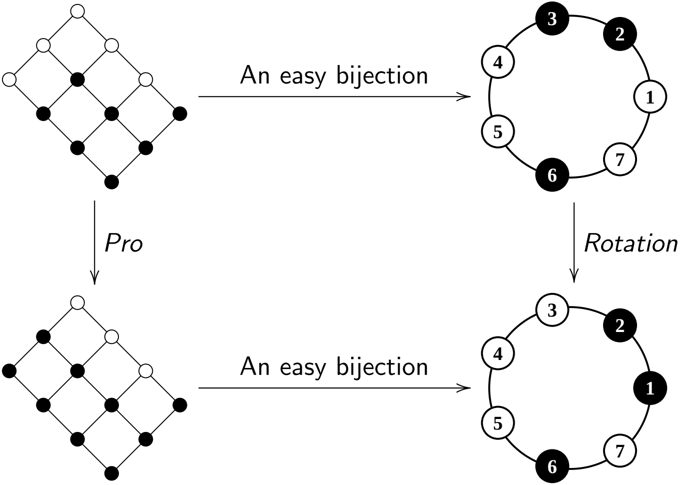
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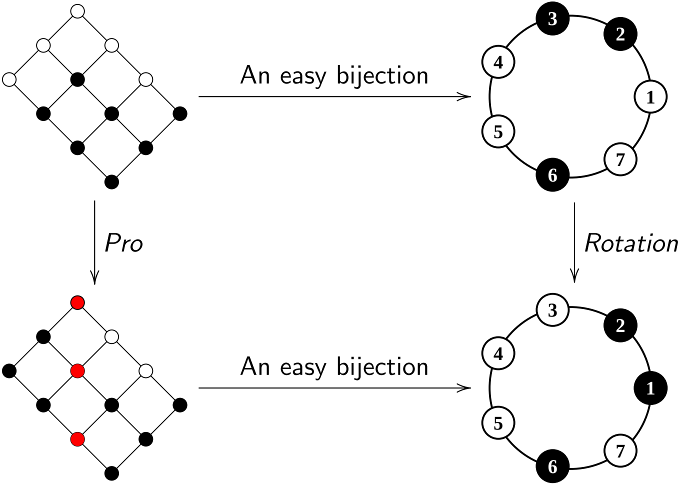
Binomial



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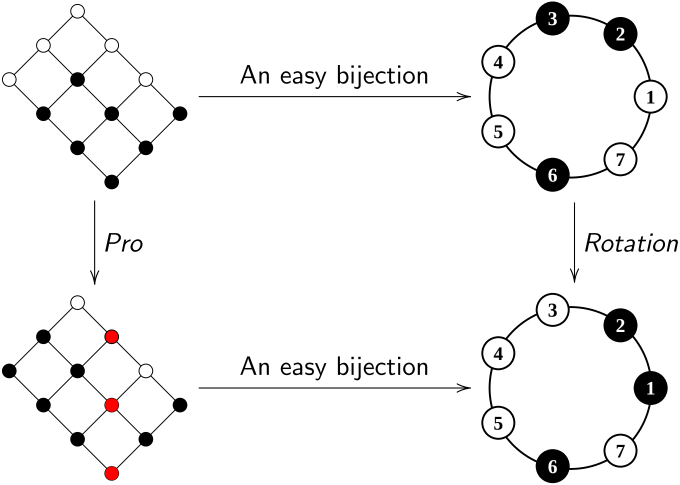

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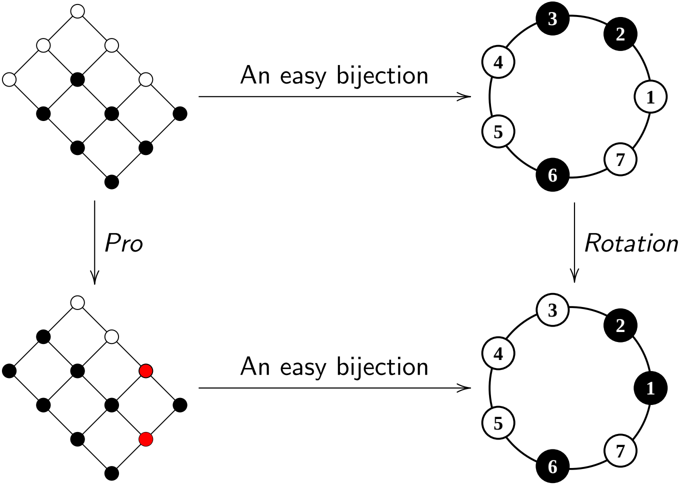
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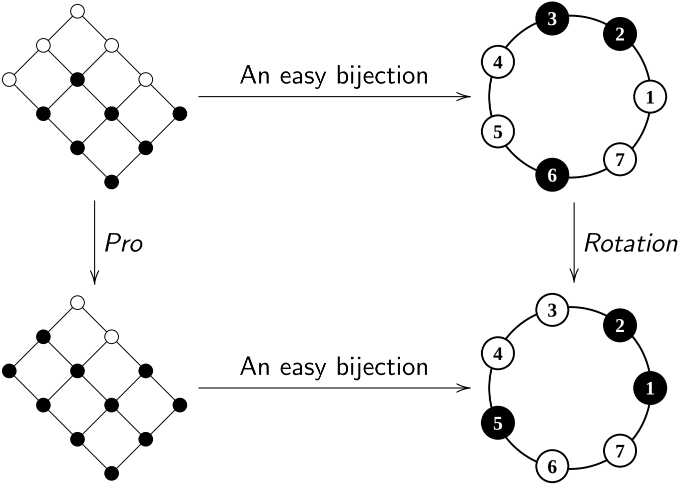

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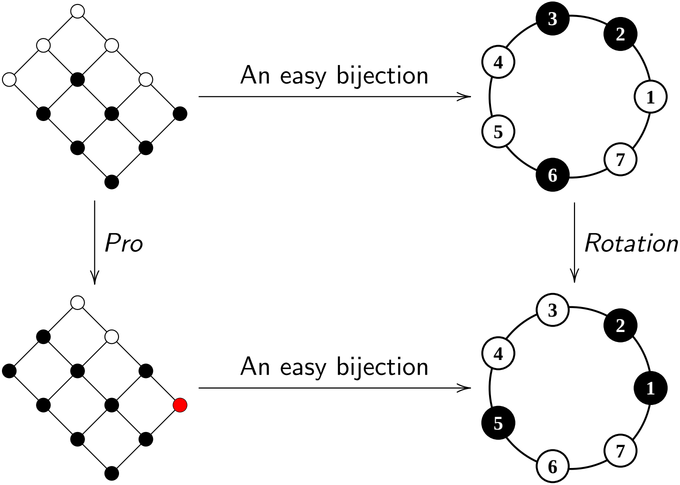

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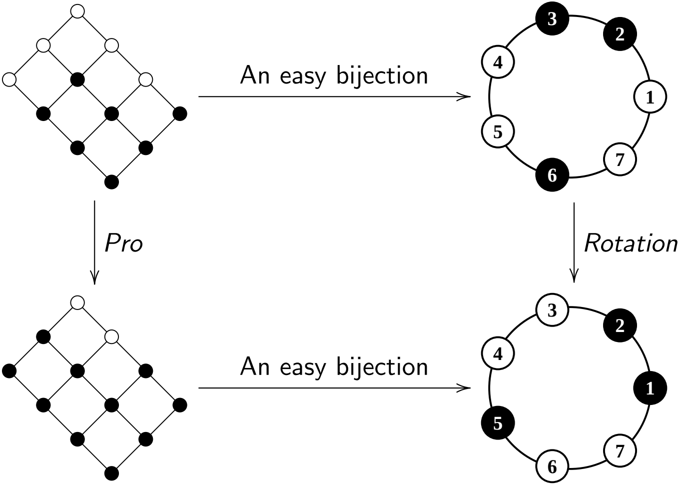

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An easy bijection
Pro
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An easy bijection
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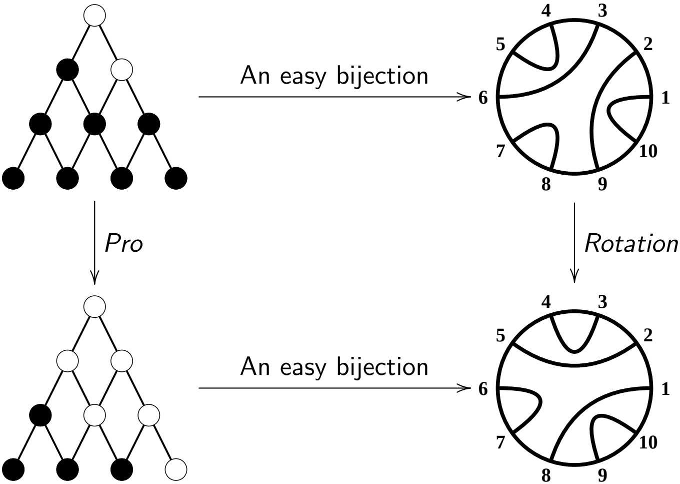
Catalan


An easy bijection
Pro
Rotation
An easy bijection
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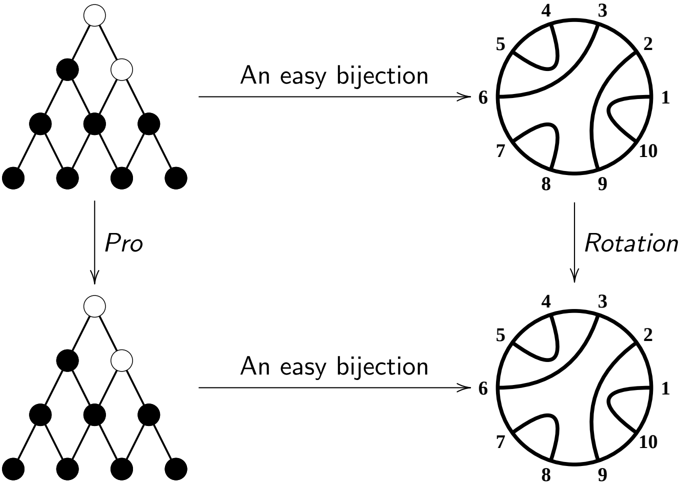
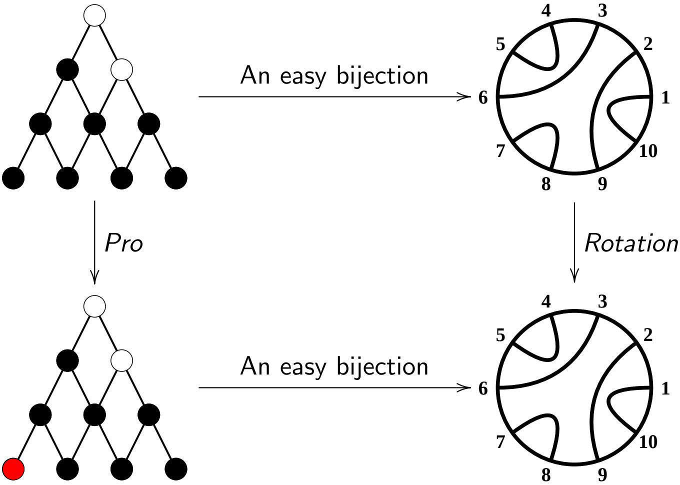
Catalan


An easy bijection
Pro
Rotation
An easy bijection
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10

Catalan



An easy bijection
Pro
Rotation
An easy bijection
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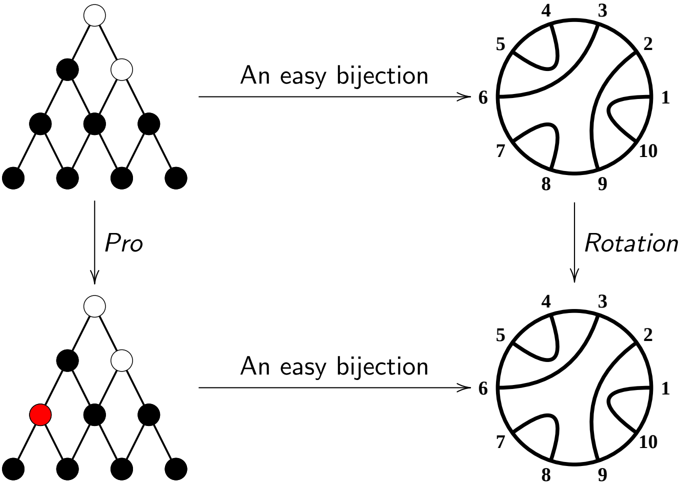
Catalan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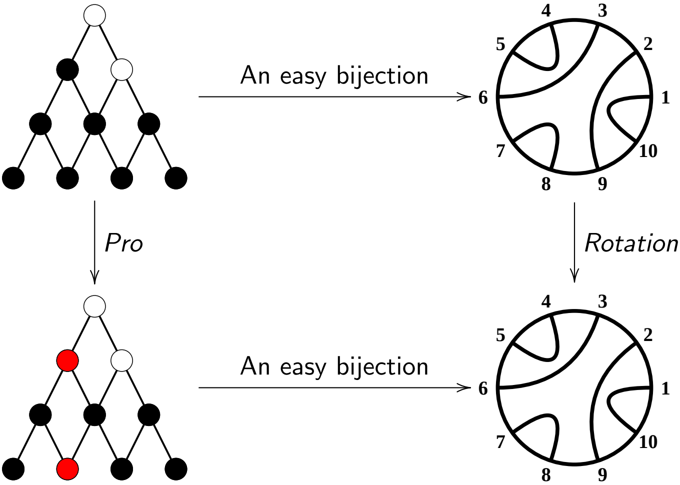
Catalan


An easy bijection
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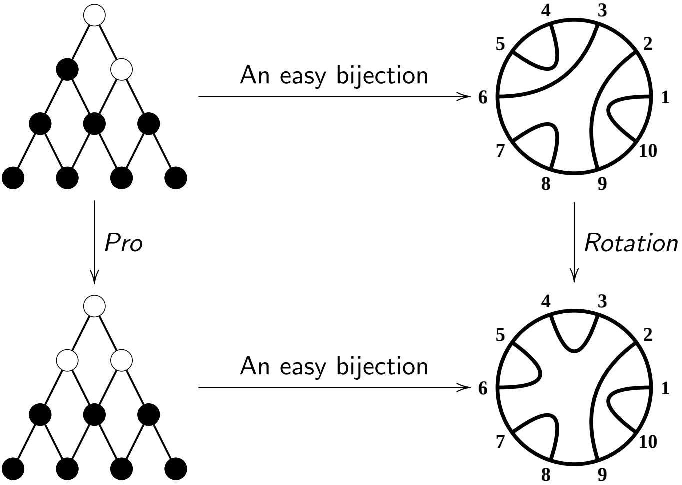
An easy bijection
4 3 5 2 6 1 7 10 8 9
4 3 5 2 6 1 7 10 8 9

Catalan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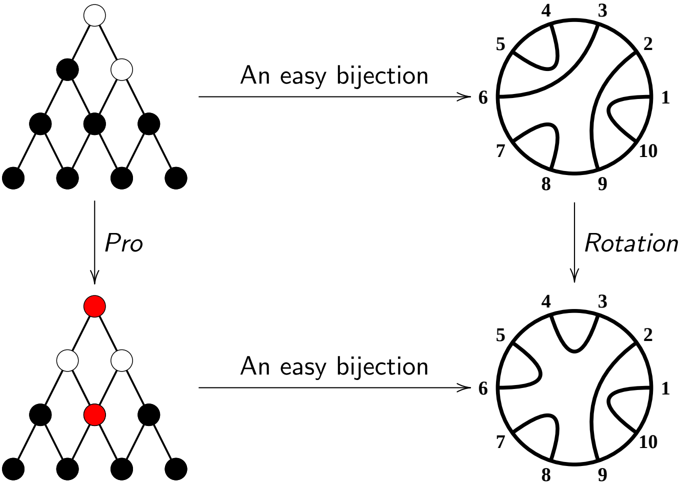
Catalan


An easy bijection
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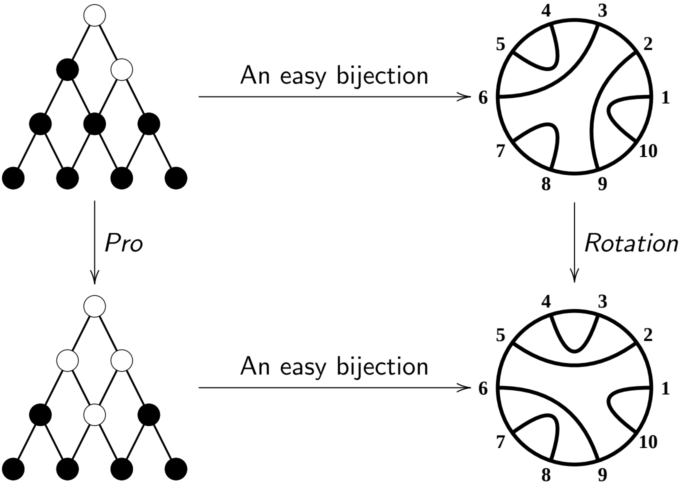
Catalan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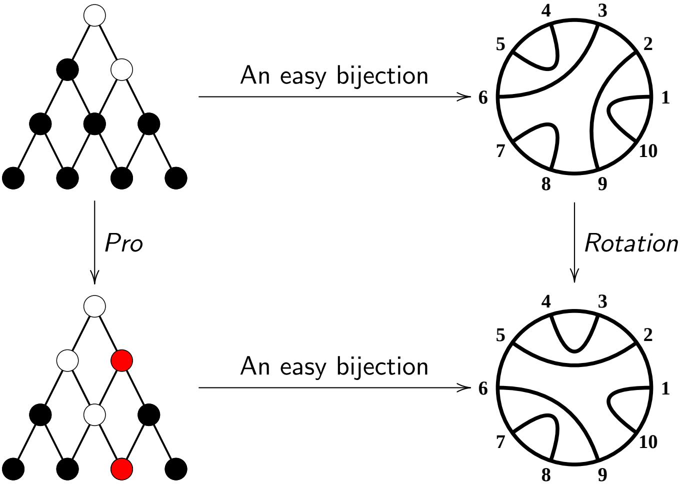
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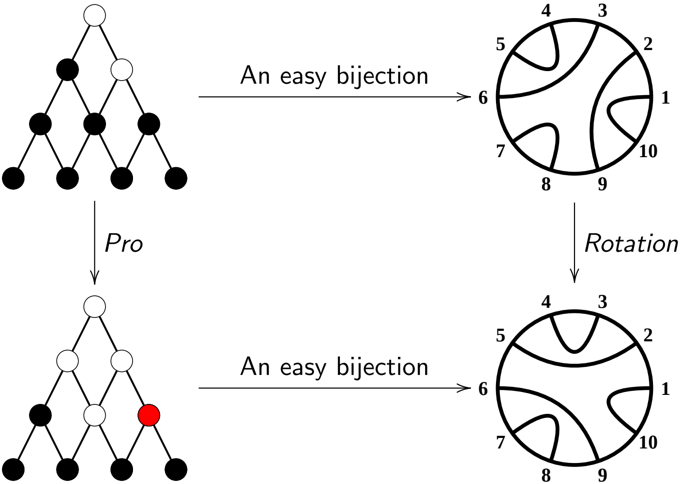
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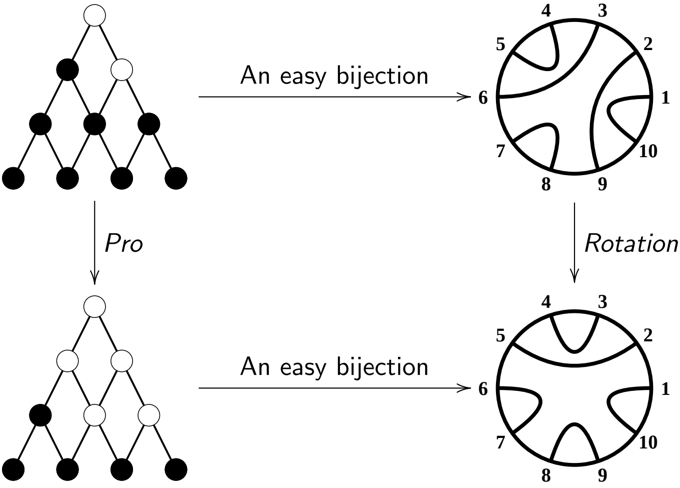

An easy bijection
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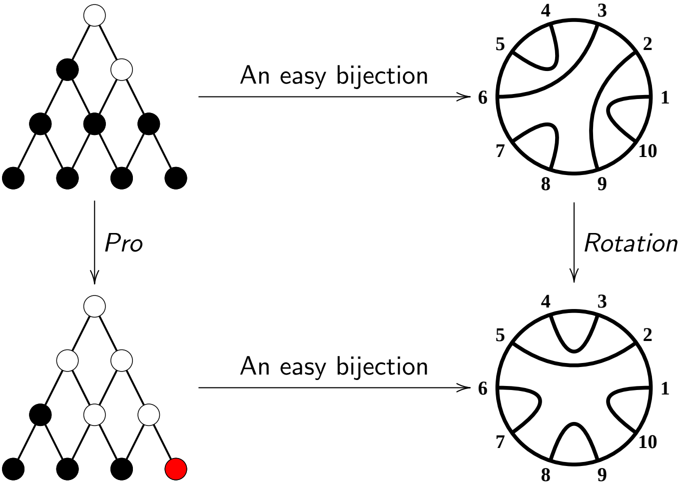
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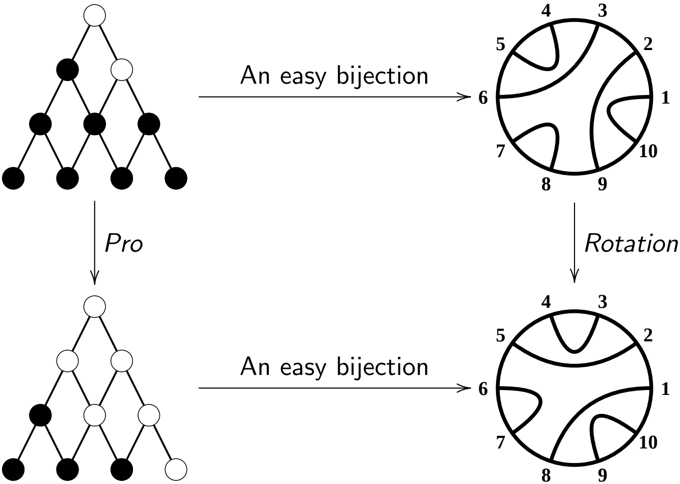
Catalan



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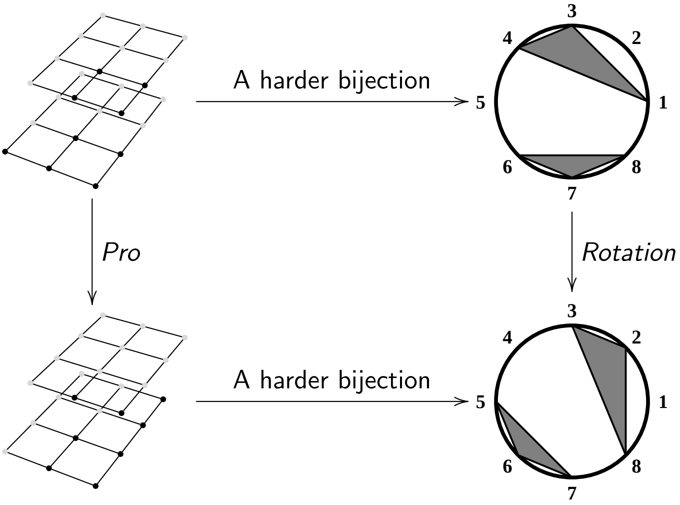
Plane Partitions (of height 2)



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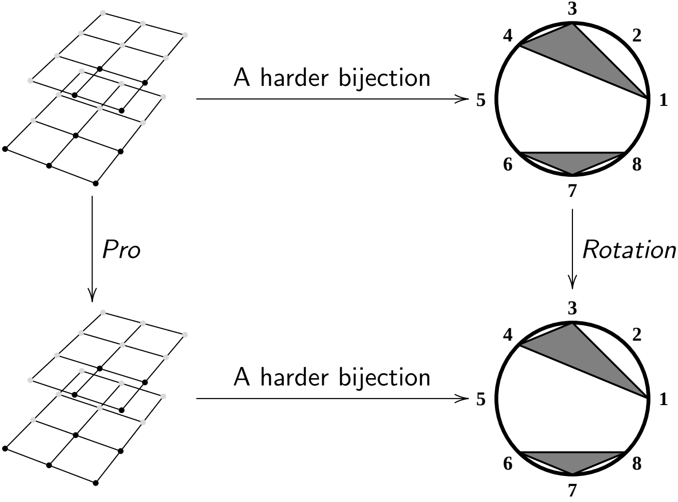

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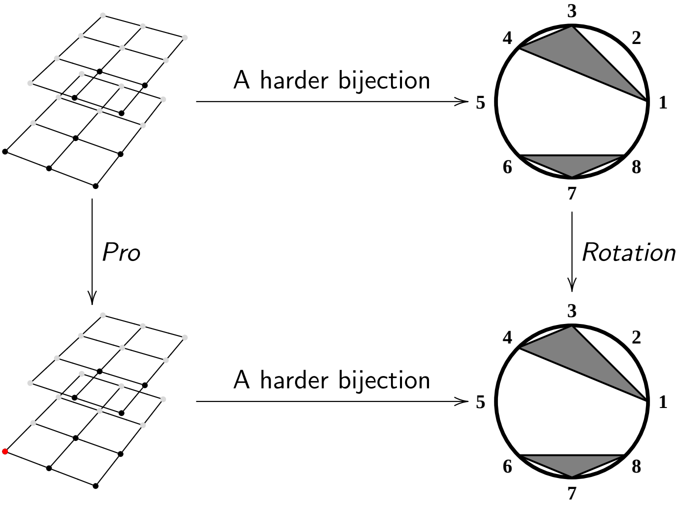

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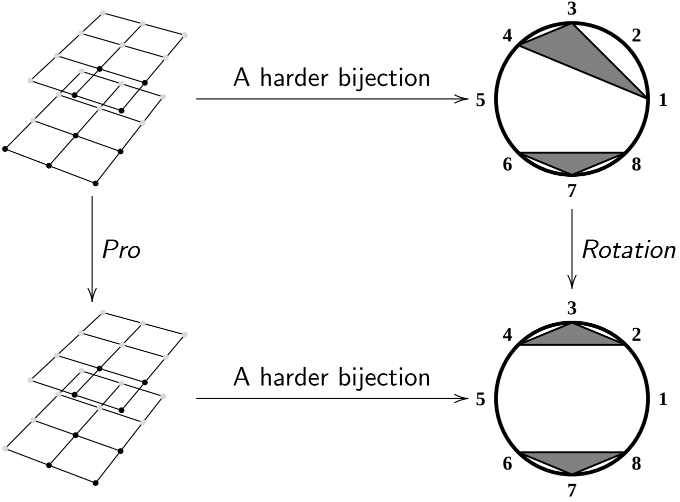

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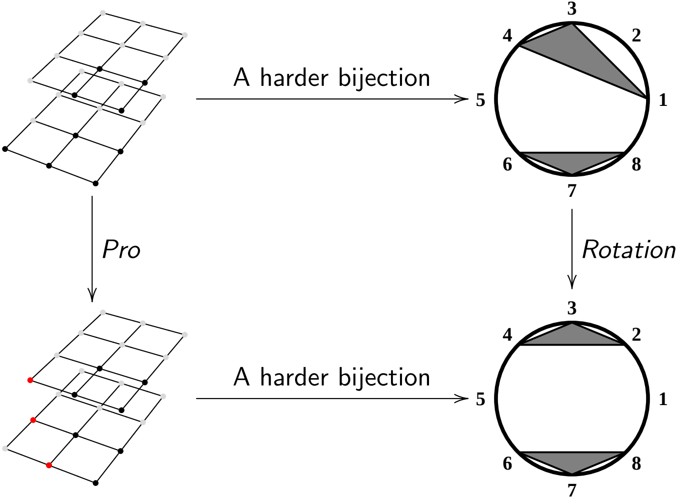

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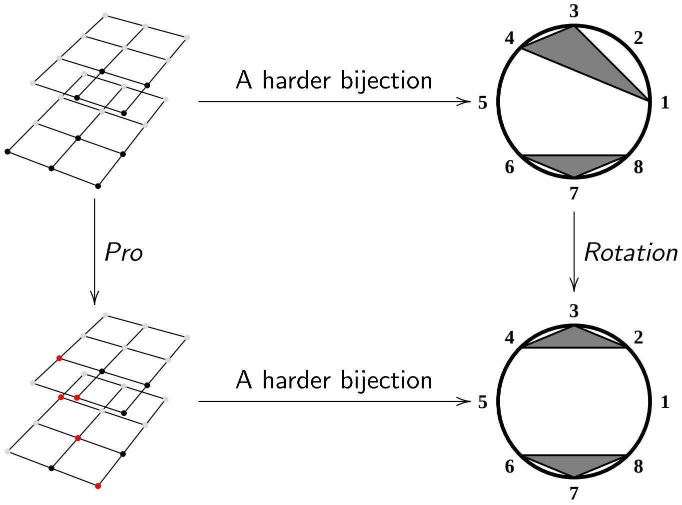

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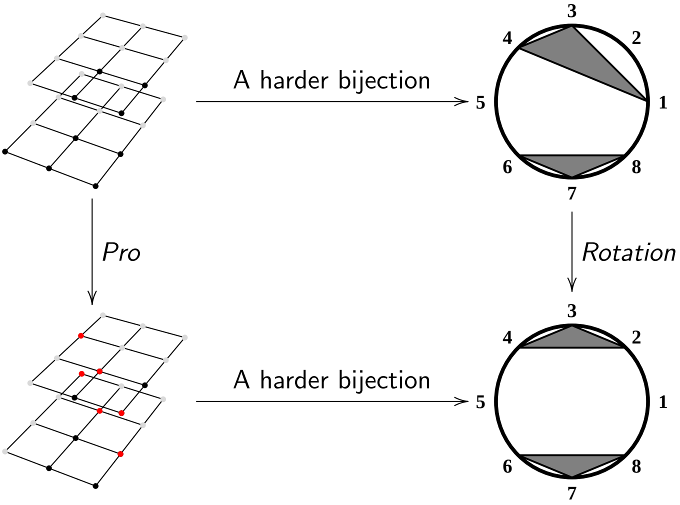

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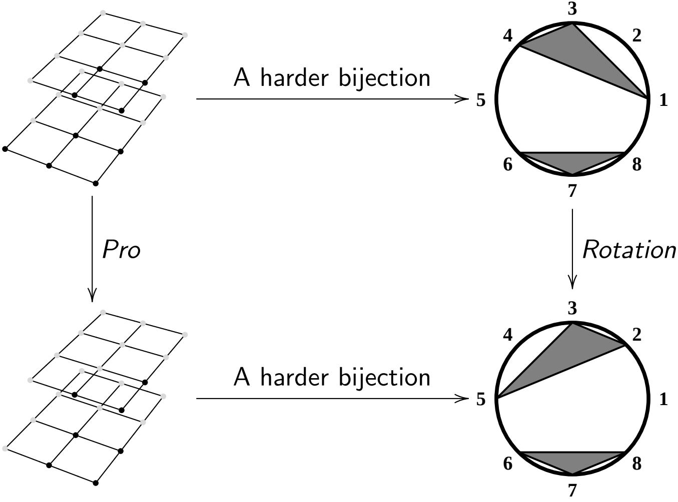

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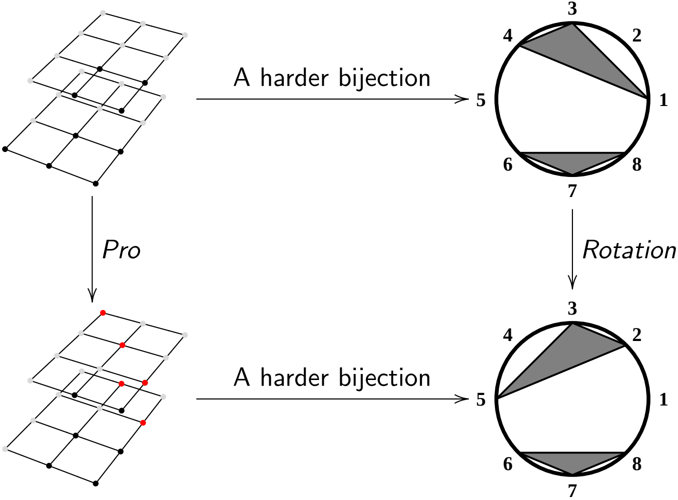

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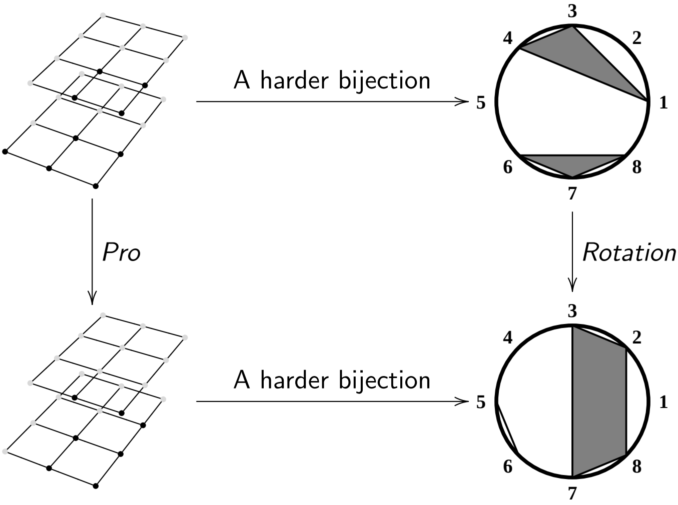

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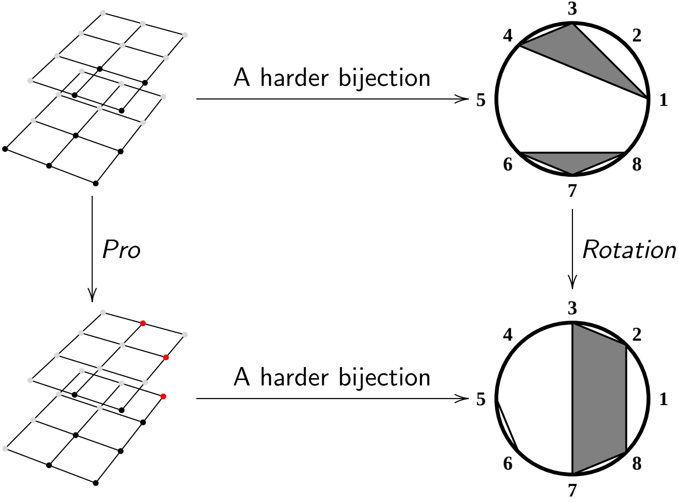

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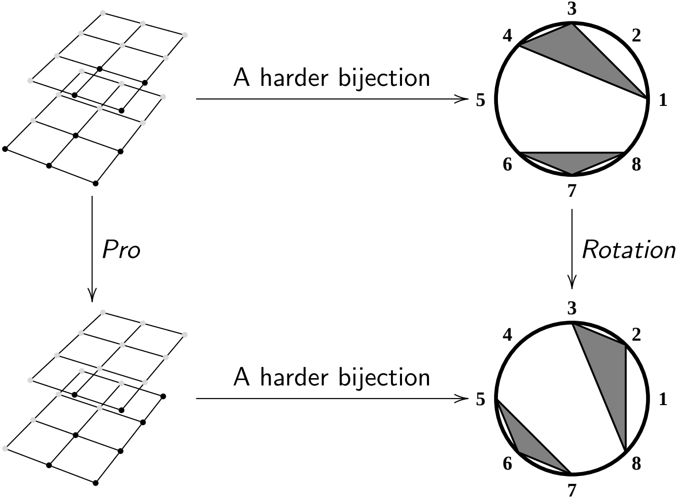

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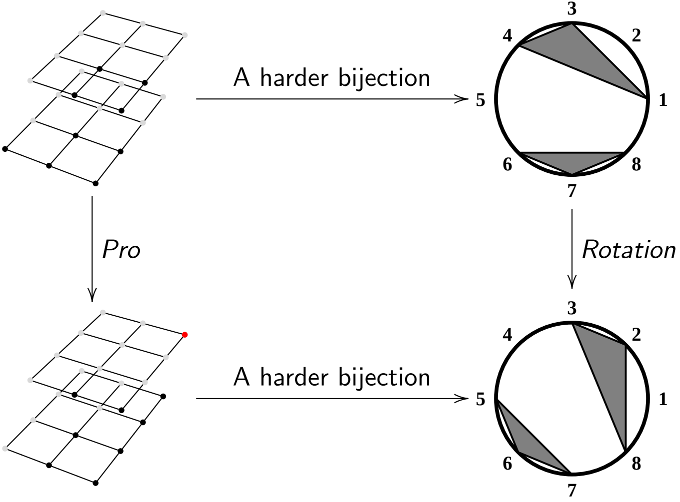

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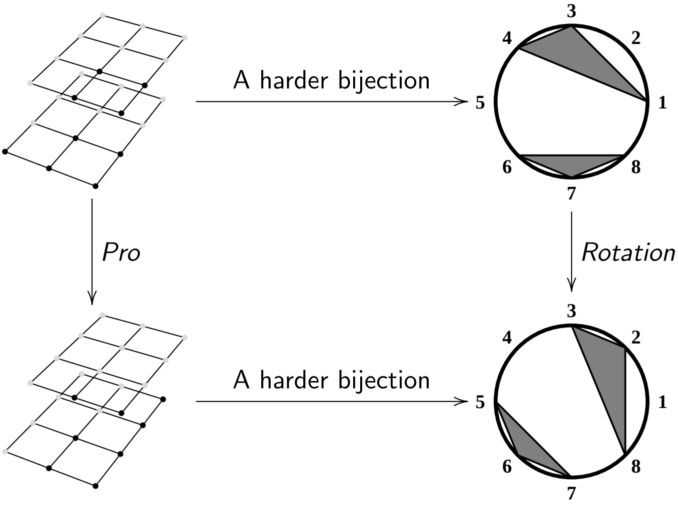

Some of our favorite order ideals are objects

Reverse Philosophy

Philosophy

"Well-behaved" elements in the toggle group should imply the existence of combinatorial models where the action of that element becomes rotation.

Types of Catalan Objects

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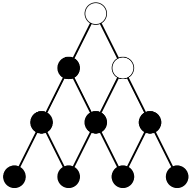
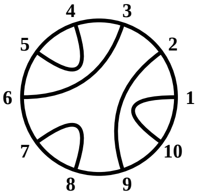
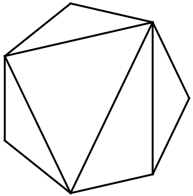
<tr><th>Nonnesting</th><th>Noncrossing</th><th>Triangulations</th></tr>
<tr><td></td><td>
</td><td></td></tr>
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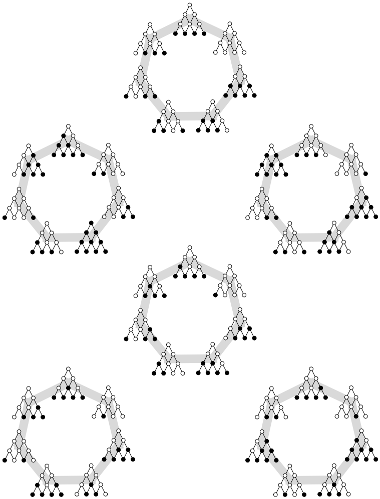
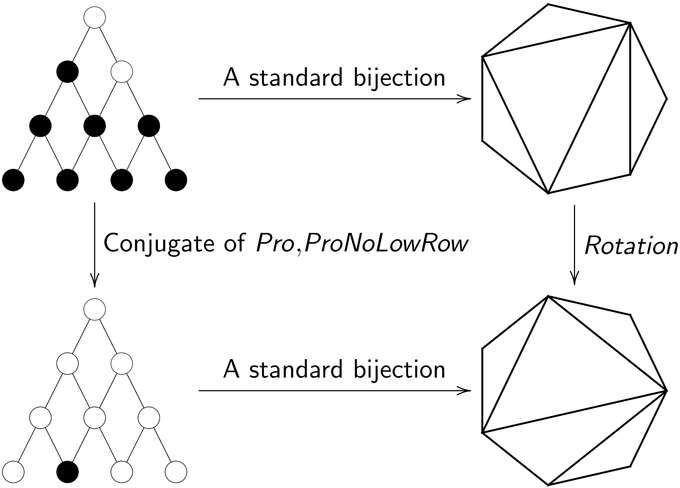
Orbits under *Pro*, then *Pro* with no lowest row.



A Conjugate Element of the Toggle Group



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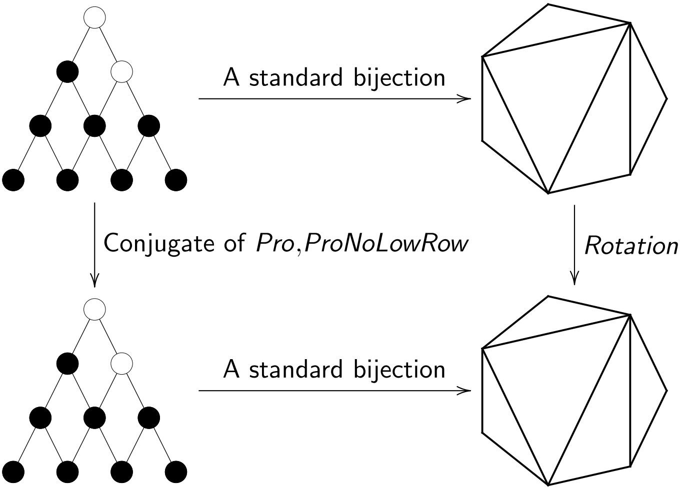

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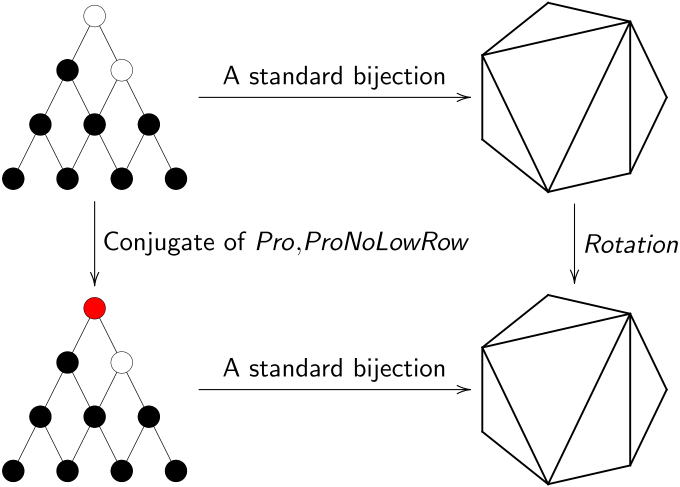

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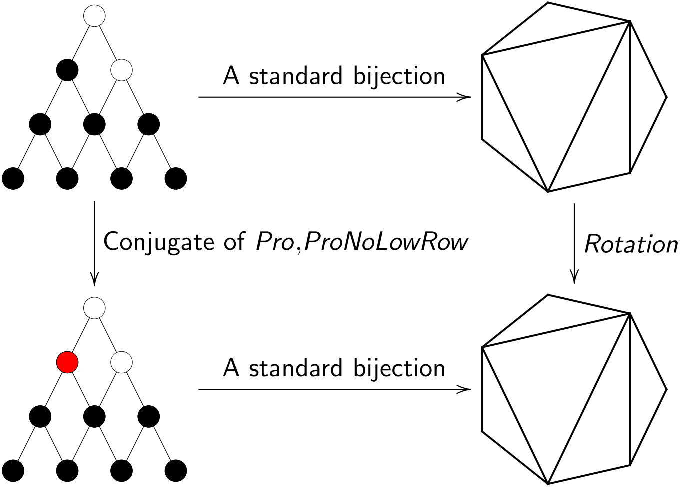

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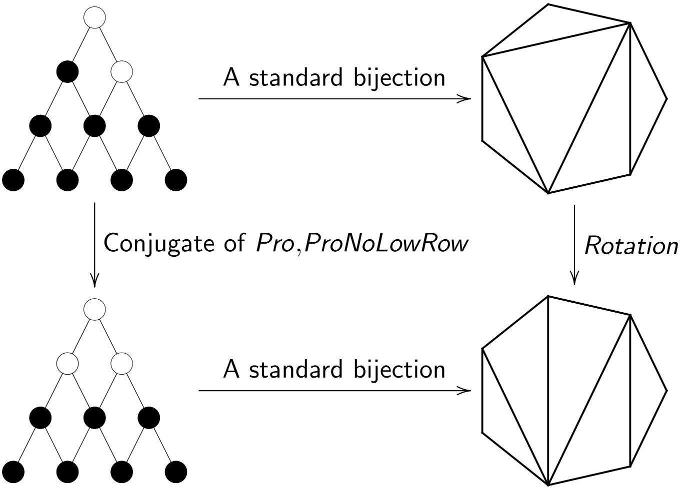

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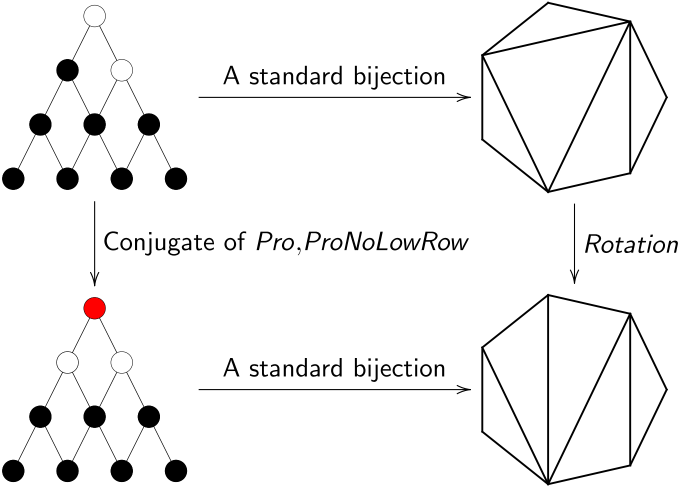

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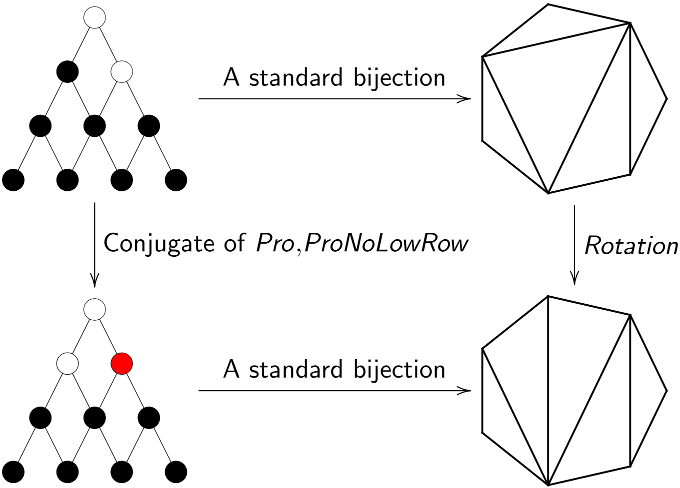

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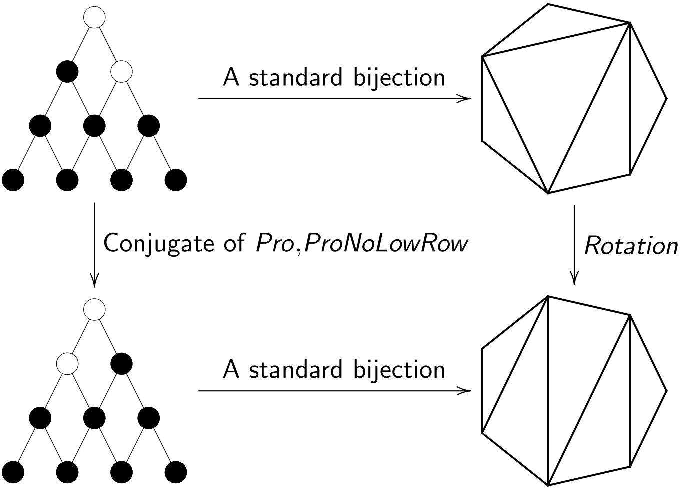

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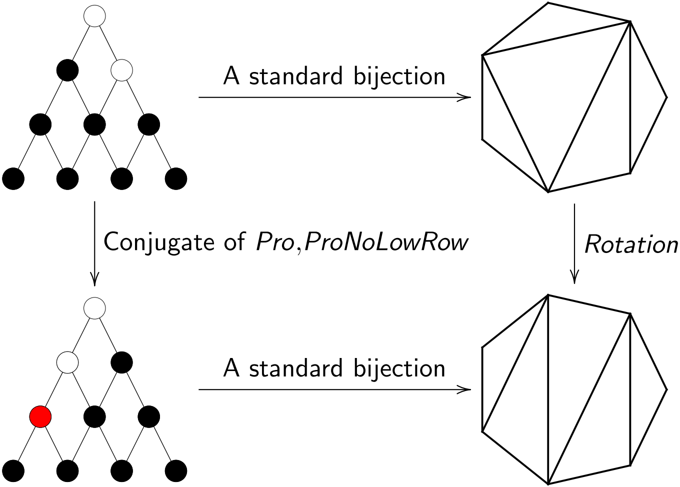

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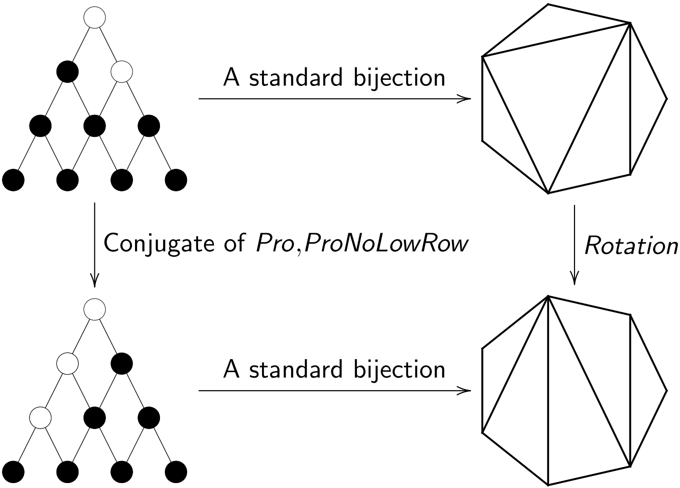

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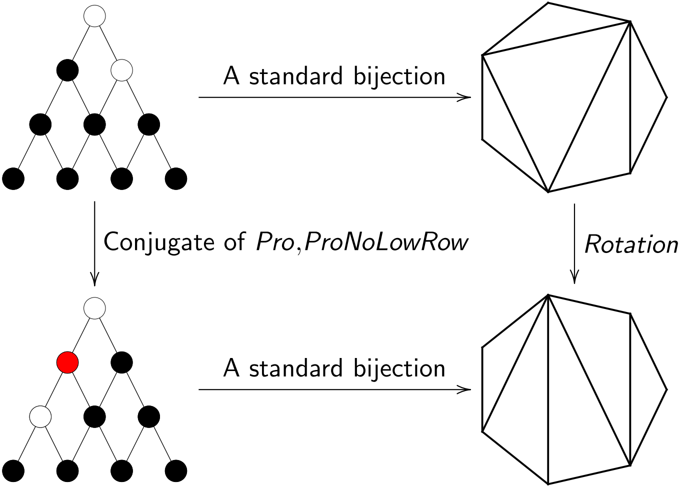

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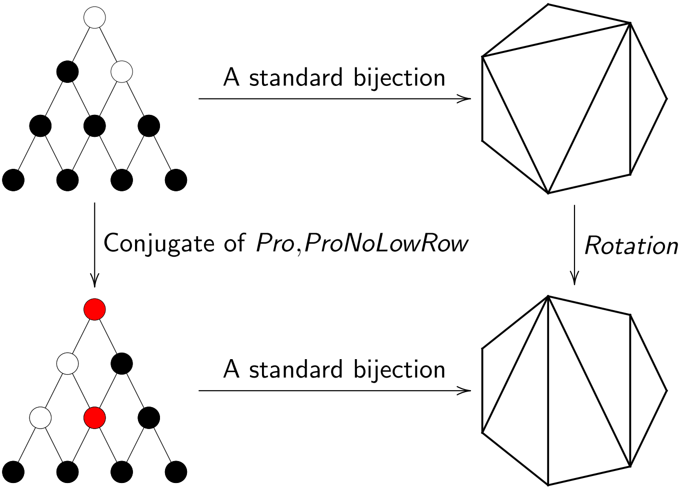

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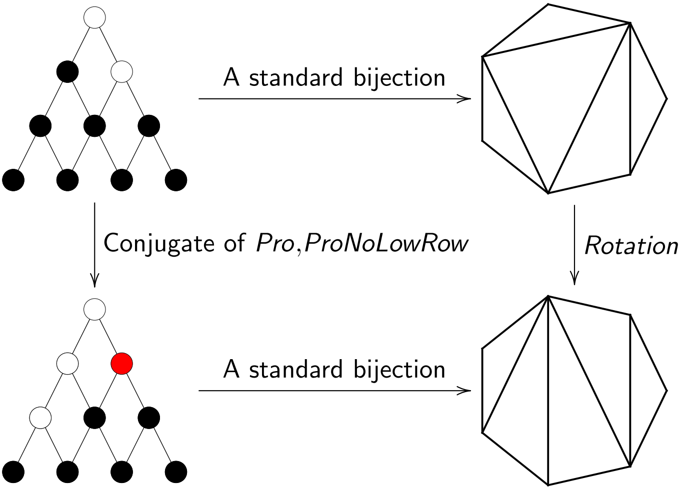

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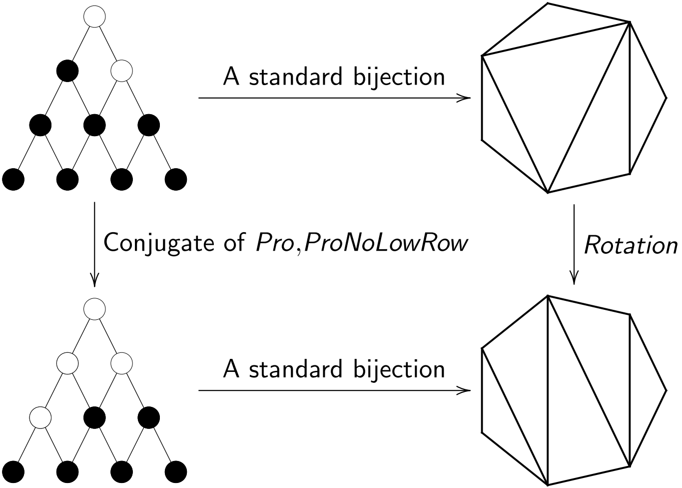

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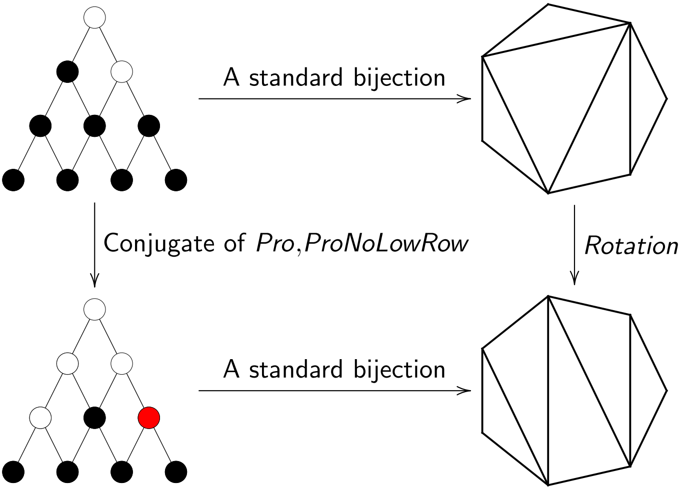

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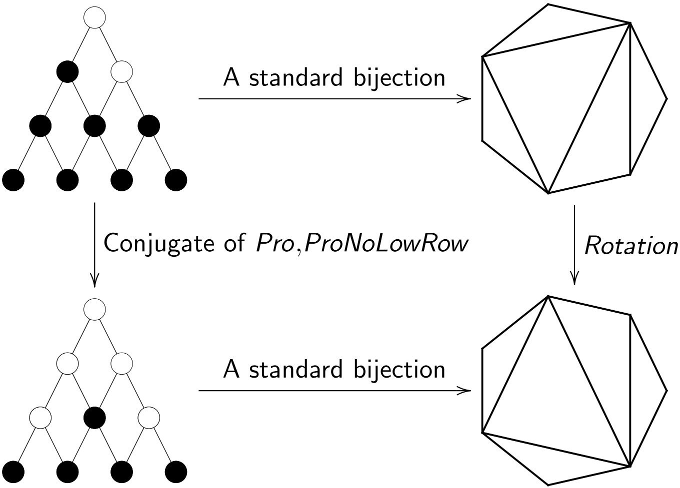

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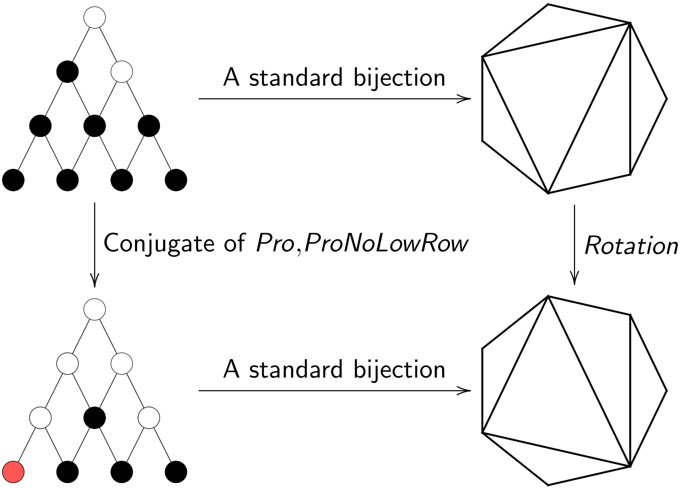

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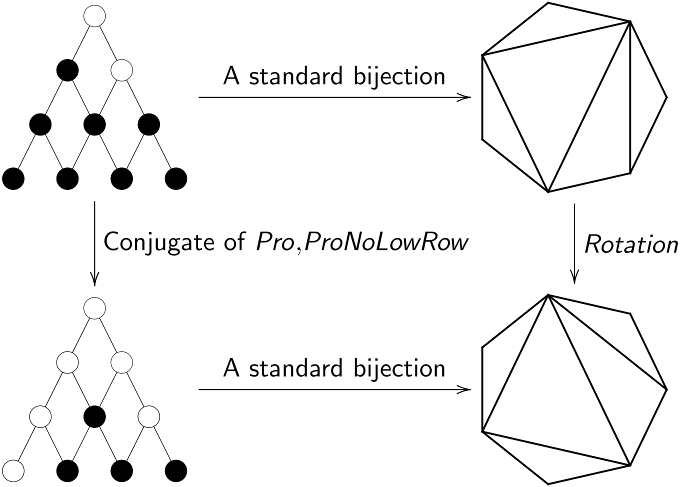

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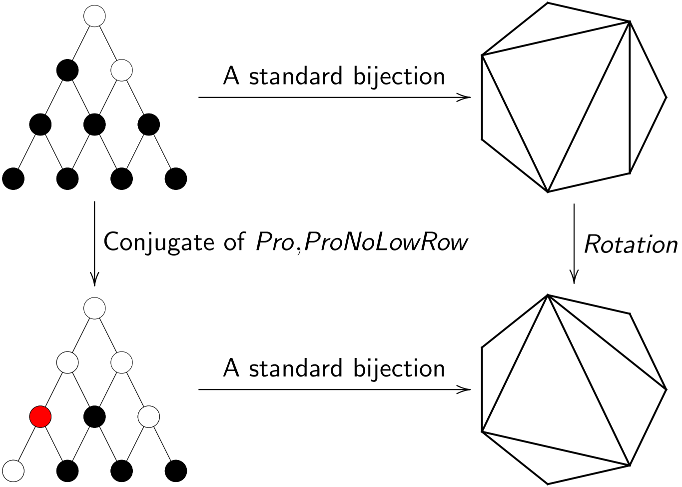

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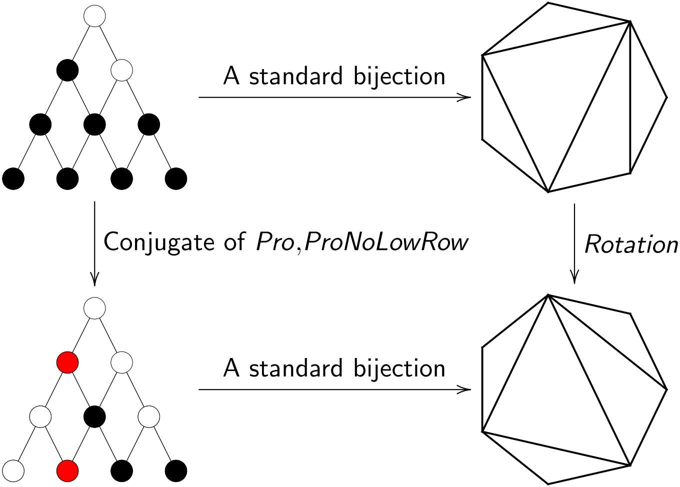

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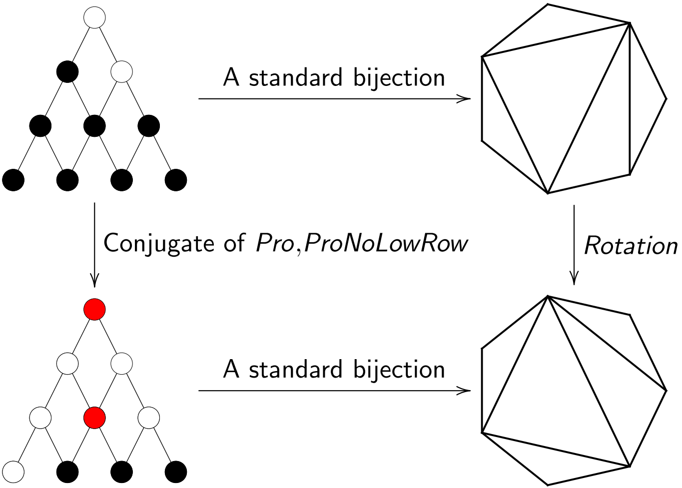

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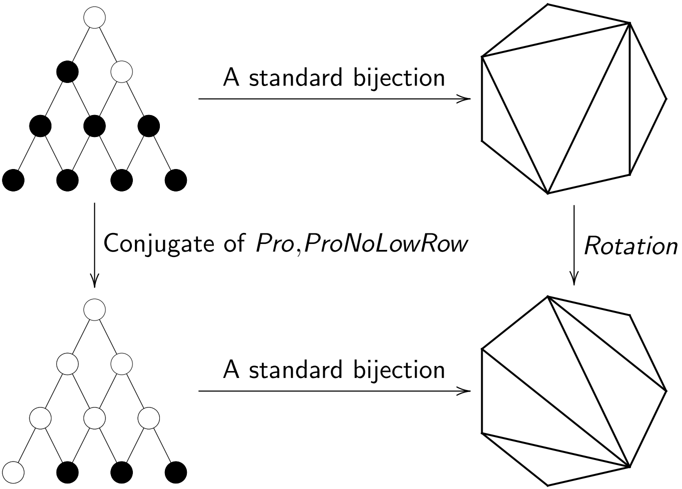

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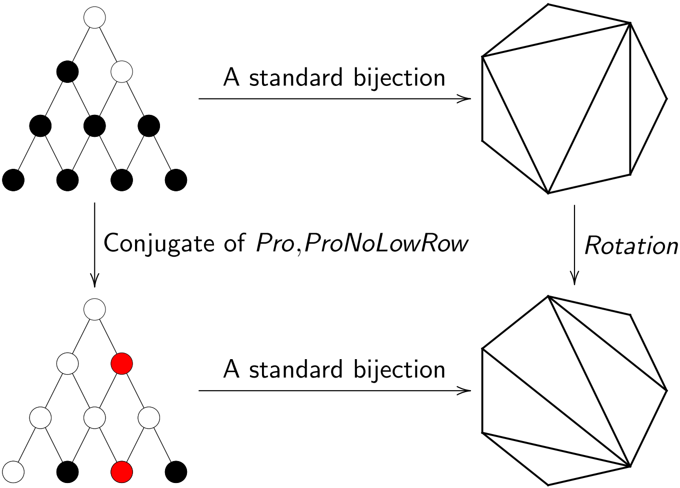

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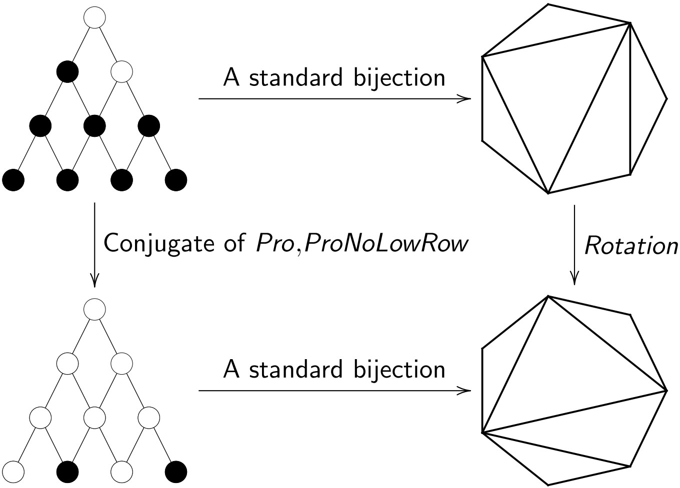

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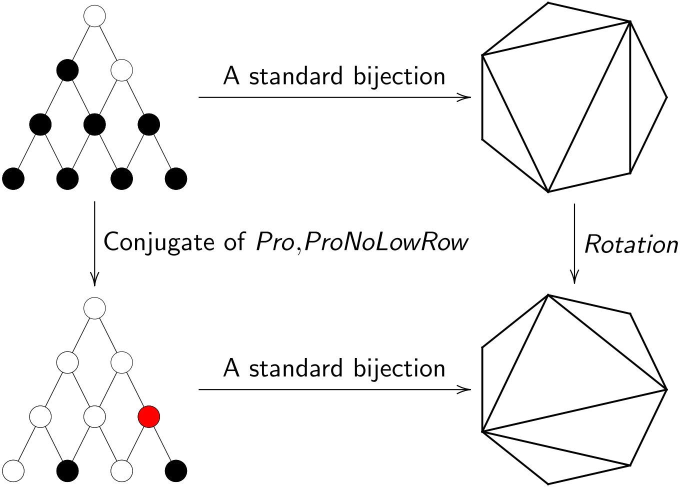

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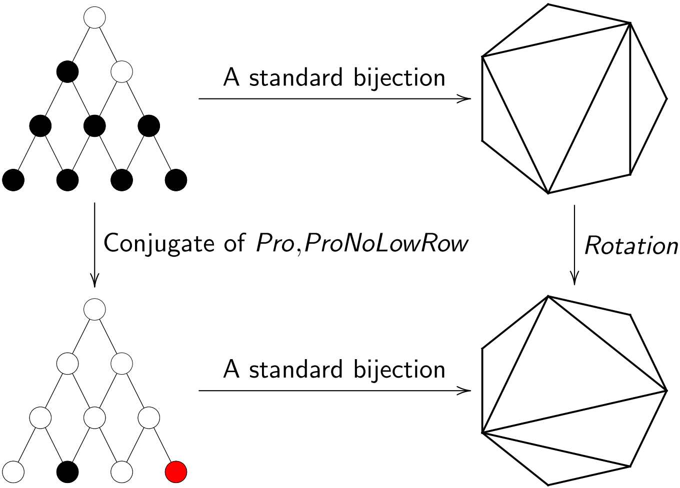

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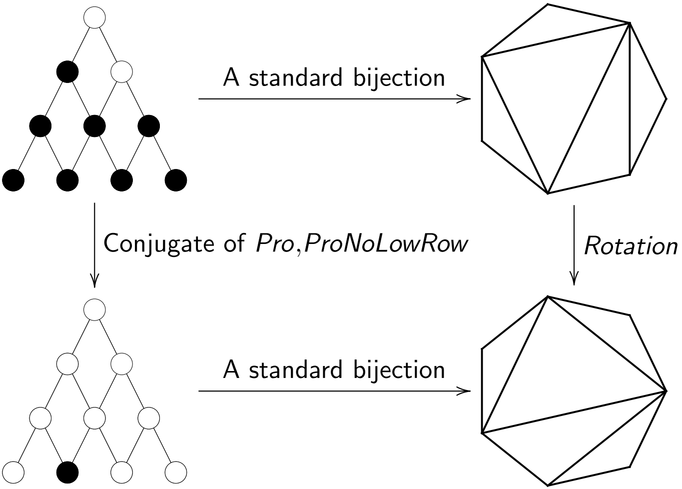

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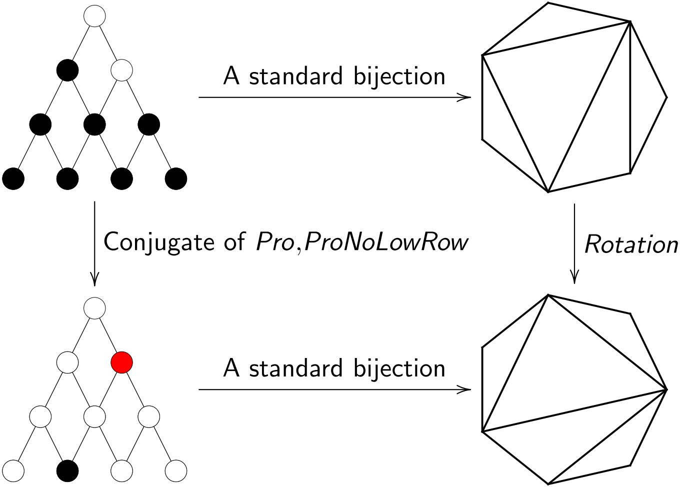

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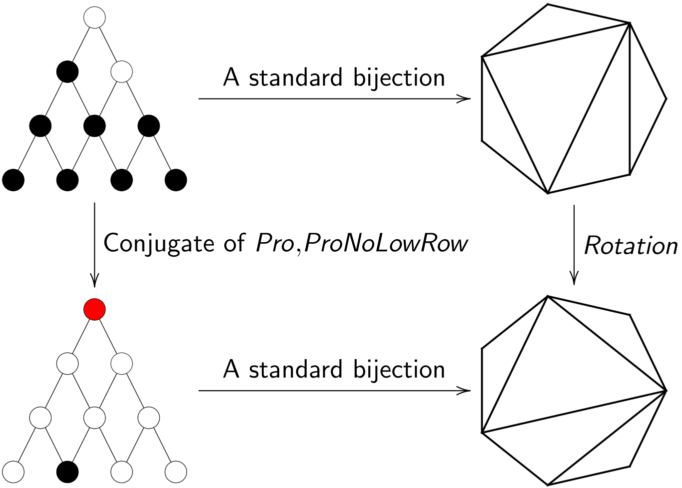

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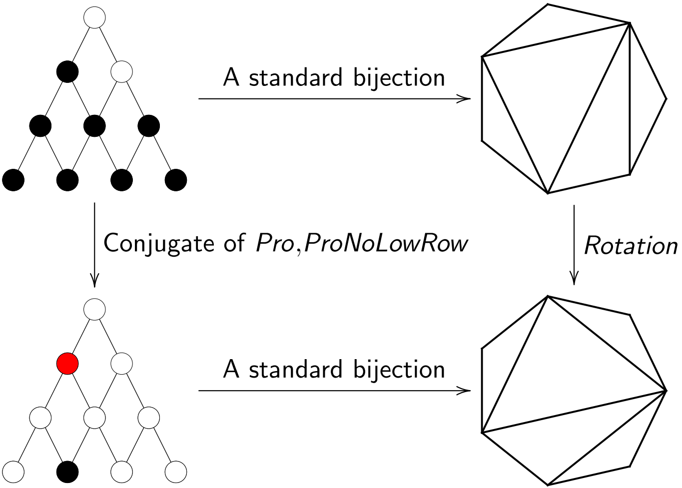

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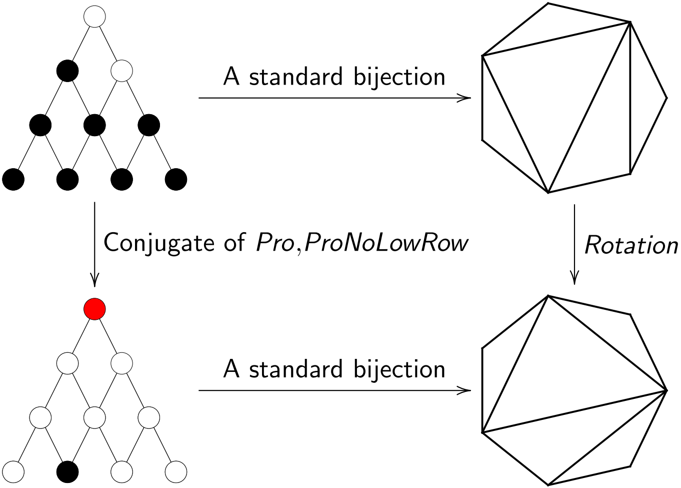

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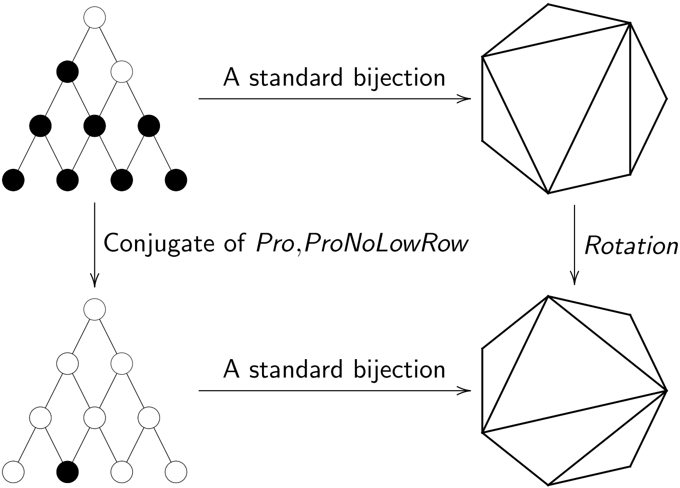
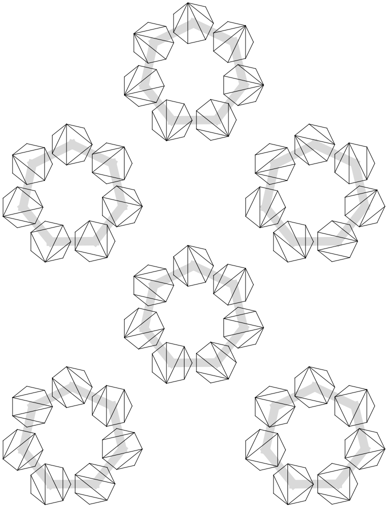
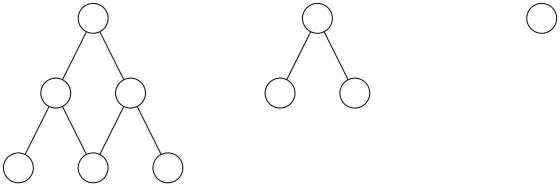


Triangulations!



Some of our favorite order ideals are objects: ASMs

Elements inherit covering relations



Nonnesting ASMs

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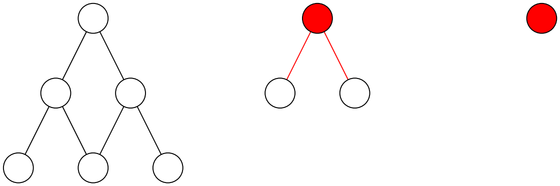

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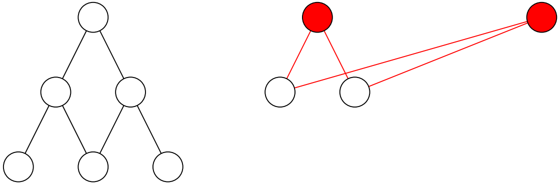

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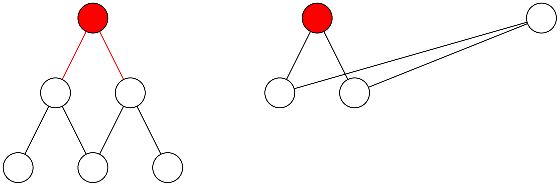

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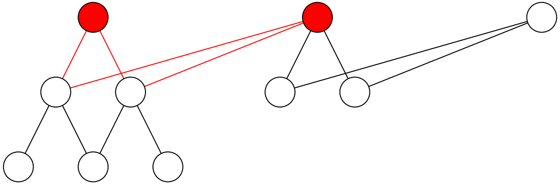

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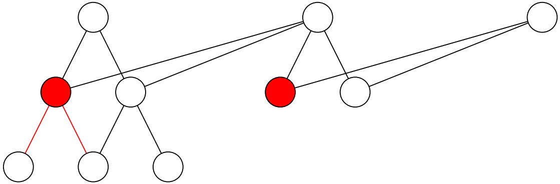

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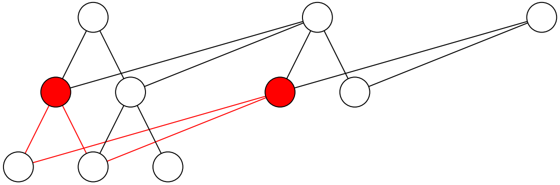

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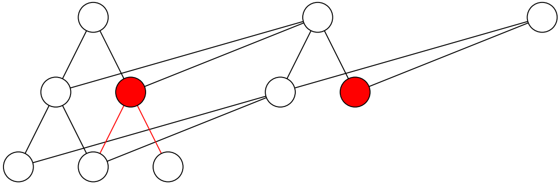
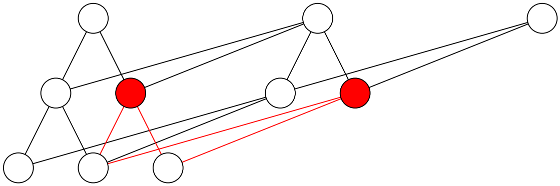


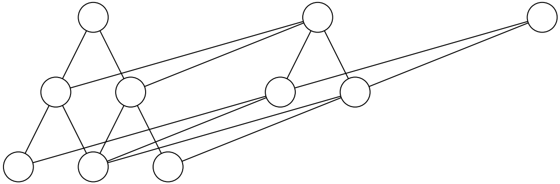
Nonnesting ASMs

Elements inherit covering relations



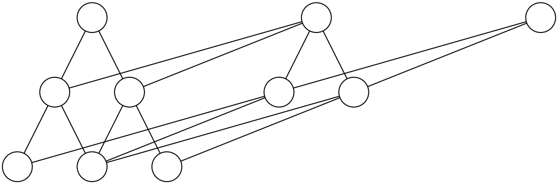
Nonnesting ASMs

Elements inherit covering relations

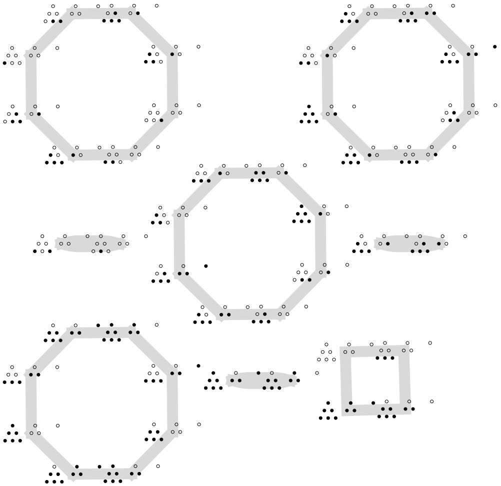


Nonnesting ASMs

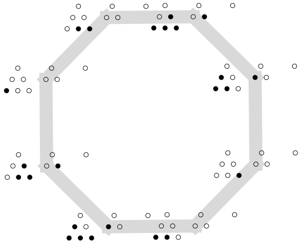
ASMs are order ideals in this poset.

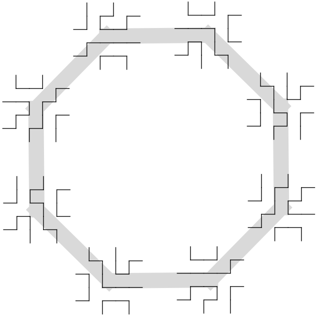


Nonnesting ASMs under Rowmotion

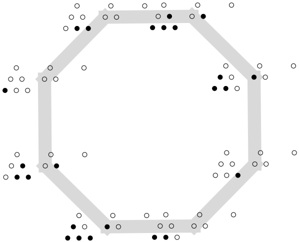


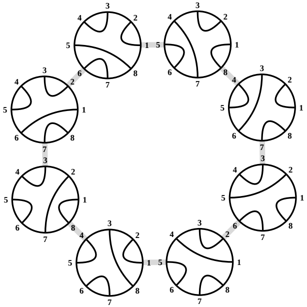
FPL: Gyration Rotates the Link Pattern



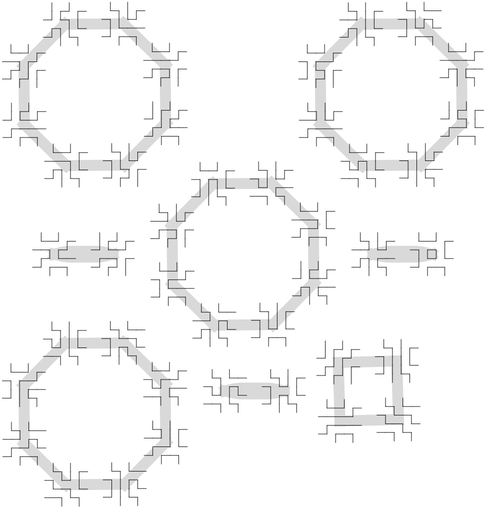


FPL: Gyration Rotates the Link Pattern



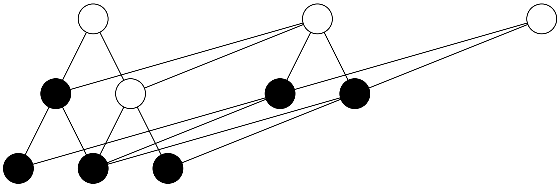


NC ASM (AKA FPL)!



Another Element of the Toggle Group...

Let *Pro* act successively in each layer.



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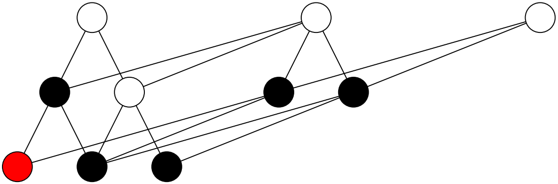

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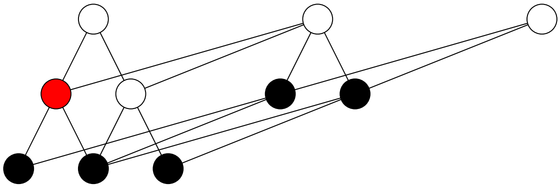

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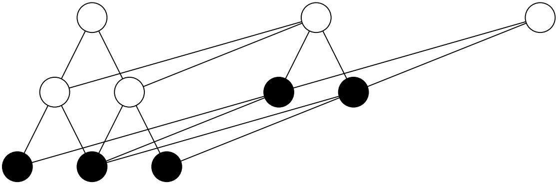

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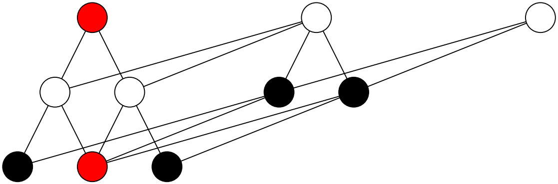

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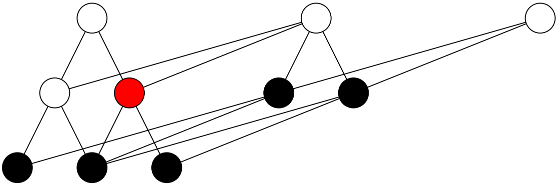

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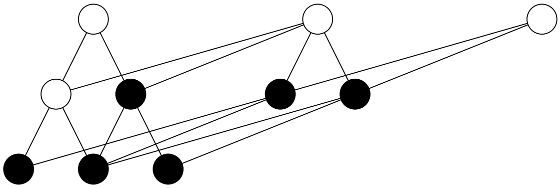

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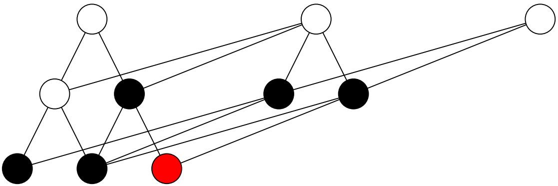

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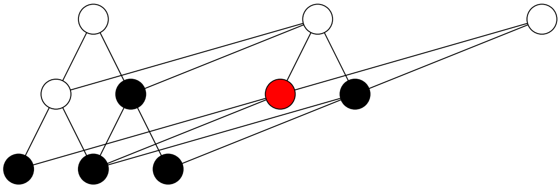

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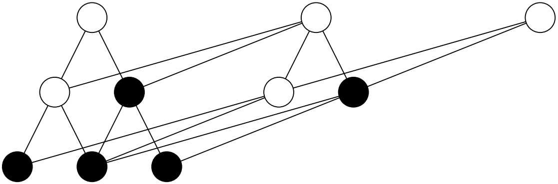

Another Element of the Toggle Group...

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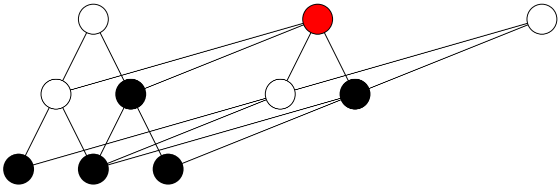
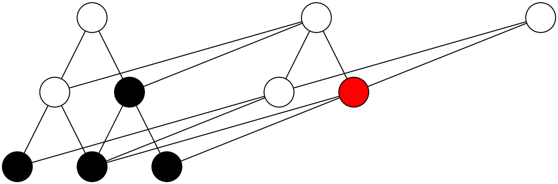


Let *Pro* act successively in each layer.



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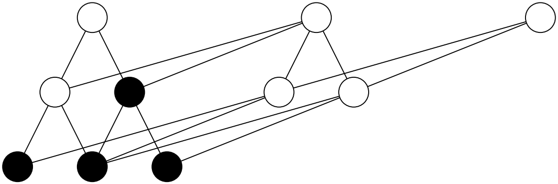

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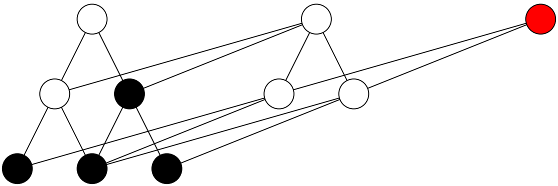

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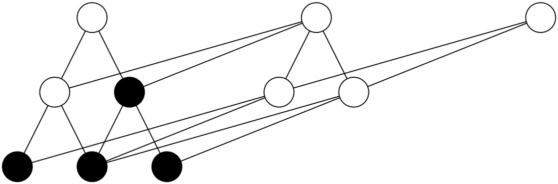
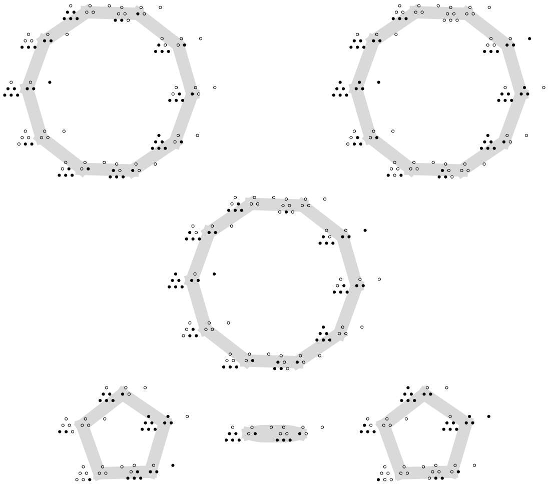
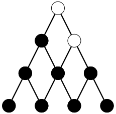
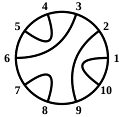
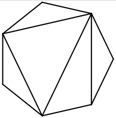
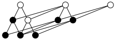
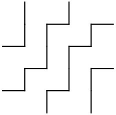


...Acting on ASMs



Question

Is there an undiscovered model for ASMs?

	Nonnesting	Noncrossing	Triangulations
Catalan			
ASM			?

Data

	ASMs under ψ		TSSCPPs under *Row*	
	Orbit Size	Number of Orbits	Orbit Size	Number of Orbits
$n = 1$	1	1	1	1
$n = 2$	2	1	2	1
$n = 3$	7	1	7	1
$n = 4$	10 5 2	3 2 1	10 5 2	3 2 1
$n = 5$	13	33	39 26 13	1 1 28
$n = 6$	16	456	112 96 80 64 48 32 24 16	1 2 2 5 23 30 2 277

Thank You!