Shellability - a nonpurist view

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Shellability is a well-known combinatorial concept that expresses a certain orderliness in the structure of a simplicial complex or a poset. Until recently it was always assumed that a shellable complex or poset is *pure* (all maximal faces or chains have equal size), for the simple reason that all observed examples of any importance were pure. Recently, in joint work with M. Wachs [5], shellability has been used to analyze the structure of some substantial nonpure examples, so it seems worthwhile to take a fresh look at the concept from this more general point of view.

Shellability has been used as a tool for many purposes: it gives information about the topology of a complex Δ and about the associated Stanley-Reisner ring $K[\Delta]$; it can be used to construct algebraic bases for homology and for $K[\Delta]$ and to compute Möbius functions of posets; it implies bounds on the f-vector of Δ which have been of importance in polytope theory and reliability theory, and the concept has also other good uses in f-vector theory.

The talk will be based on the following three topics:

1. A general review of the basic aspects of shellability with special attention given to what can be said in the nonpure case. (This is joint work with M. Wachs [5].)
2. A description of how nonpure lexicographic shellability is used to analyze the intersection lattices of the "k-equal" subspace arrangements of types A, B and D. (Joint work with M. Wachs [5] and B. Sagan [6].)
3. Presentation of a new f-vector theorem [7] that uses nonpure shellability at one step in the proof. This gives a common generalization of the theorems of Kruskal-Katona, Stanley and Björner-Kalai, and also contains a characterization of the f-vectors of complexes whose Stanley-Reisner ring has depth $\geq k$, for some integer k.

References and reading list:

The papers [5, 6, 7] on which the talk is based are still being written (January 1994) but will hopefully be available at the time of the meeting. The other items are places where one can read about pure shellability in the sense in which it will be discussed here.

[1] A. Björner, A.M. Garsia and R.P. Stanley, *An introduction to Cohen-Macaulay partially ordered sets*, in " Ordered Sets" (ed. I. Rival), Reidel, Dordrecht, 1982, pp. 583-615.

[2] A. Björner and M. Wachs, *On lexicographically shellable posets*, Trans. Amer. Math. Soc. 277 (1983) 323-341.

[3] A. Björner and G. Kalai, *On f-vectors and homology*, in "Combinatorial Mathematics: Proceedings of the Third Intern. Conf., New York 1985" (ed. G. Bloom, R. Graham and J. Malkevitch), Annals of N.Y. Acad. Sci., Vol. 555, New York Acad. Sci., 1989, pp. 63-80.

[4] A. Björner, *The homology and shellability of matroids and geometric lattices*, in "Matroid Applications" (ed. N. White), Cambridge Univ. Press, 1992, pp. 226-283.

[5] A. Björner and M. Wachs, *Shellable nonpure complexes and posets*, in preparation.

[6] A. Björner and B. Sagan, *The "k-equal" subspace arrangements of types B and D*, in preparation.

[7] A. Björner, *A common generalization of several f-vector theorems*, in preparation.