Combinatorial problems connected to the homology of Lie algebras

Philip Hanlon
Department of Mathematics
University of Michigan
Ann Arbor, MI 48109

In this talk we give a friendly introduction to the subject of Lie algebra homology and describe a number of open problems. Throughout, the talk will emphasize problems and results of a combinatorial nature. Almost all of the talk (and all of the important information) will be understandable to those with a strong background in linear algebra.

References:

Basics about Lie algebras and Lie algebra homology:

[1] J.E.Humphreys, "Introduction to Lie Algebras and Representation Theory", Springer Graduate Texts in Mathematics #9.

[2] J.L.Koszul, "Homologie et cohomologie des algèbres de Lie", Bull. Math. Soc. France, **78** (1950) 65-127 (the seminal work on the subject).

[3] Most standard texts in homological algebra have some introduction to Lie algebra homology, eg., Rotman, Hilton & Stammbach, Cartan & Eilenberg.

More directly relevant to this talk:

[4] B. Kostant, "Lie algebra cohomology and the generalized Borel-Weil Theorem", Ann. of Math. **74** (1961) 329-387 (very technical but a dynamite paper).

[5] H. Garland and J. Lepowsky, "Lie algebra homology and the Macdonald-Kac formulas", Inv. Math. **34** (1976) 37-76 (same comment as [4]).

[6] J. Stembridge, "First layer formulas for characters of $SL(n, \mathbb{C})$", Trans. AMS **299**, #1 (1987) 319-350.

[7] I.G. Macdonald, "Some conjectures for root systems", SIAM J. Math. Anal. **13** (1982) 988-1007.

[8] J.L. Loday and D. Quillen, "Cyclic homology and the Lie algebra homology of matrices", Comm. Math. Helv. **59** (1984) 565-591 (see also the new book, "Cyclic Homology" by J.L. Loday, Springer Grundlehren der mathematischen Wissenschaften #301).

[9] R.P. Stanley, "The stable behavior of some characters of $SL(n, \mathbb{C})$", J. Linear and Multilinear Alg. **16** (1984) 3-27.

[10] P. Hanlon, "Cyclic homology and the Macdonald conjectures", Inv. Math. **86** (1986) 131-159.

[11] P. Hanlon, "Some conjectures and results concerning the homology of nilpotent Lie algebras", Adv. Math. **84**, #1 (1990) 91-134.