Modular representations, crystal bases, and the combinatorics of partitions

Jean-Yves Thibon
Institut Gaspard Monge
Université de Marne-la-Vallée
5 Boulevard Descartes, Champs-sur-Marne
77454 Marne-la-Vallée cedex 2, France

Abstract: It has been realized in the past few years that the same combinatorics was underlying two apparently unrelated subjects: the representation theory of symmetric groups over a field F of finite characteristic n, and the affine Lie algebra $\widehat{sl}_n$ [LLT96]. A precise connection between these theories can be formulated as follows: the direct sum of the complexified Grothendieck groups $\bigoplus_{m\geq 0} G_0(F[S_m])$, endowed with some refined restriction and induction operators originally defined by Robinson [Ro61], build up the basic representation of $\widehat{sl}_n$.

The representation theory of the type A Hecke algebra $H_m(\zeta)$, for ζ a primitive nth root of unity also involves the same combinatorics, and there, n need not be a prime. Consideration of the global crystal basis (canonical basis) of the basic representation of $U_q(\widehat{sl}_n)$ led to a conjectural description of the decomposition matrices of Hecke algebras at roots of unity [LLT96]. This conjecture was subsequently proven in a more general form by Ariki [Ar]. A proof was also announced by Grojnowski, relying on the results of [Groj]. Another extension [LT96] provided a conjectural description of the decomposition matrices of q-Schur algebras at roots of unity. It was given in terms of a new canonical basis of the Fock space representation of $U_q(\widehat{gl}_n)$, constructed by means of ribbon tableaux (see [LLT97]).This conjecture has been settled recently by Varagnolo and Vasserot [VV98].

Other generalizations and applications can be found in [FLOTW1,FLOTW2,LT97].

Reference

[Ar] S. Ariki, *On the decomposition numbers of the Hecke algebra of $G(n,1,m)$*, J. Math. Kyoto Univ. **36** (1996), 789–808.

[FLOTW1] O. Foda, B. Leclerc, M. Okado, J.-Y. Thibon and T. Welsh, *RSOS models and Jantzen-Seitz representations of Hecke algebras at roots of unity*, Lett. Math. Phys. (1998).

[FLOTW2] O. Foda, B. Leclerc, M. Okado, J.-Y. Thibon and T. Welsh, *Branching functions of $A^{(1)}_{n-1}$ and Jantzen-Seitz problem for Ariki-Koike algebras*, preprint, 1997.

[Groj] I. Grojnowski, *Representations of affine Hecke algebras (and affine quantum GL_n) at roots of unity*, Internat. Math. Res. Notices **5** (1994), 215–217.

[LLT96] A. Lascoux, B. Leclerc and J.-Y. Thibon, *Hecke algebras at roots of unity and crystal bases of quantum affine algebras*, Commun. Math. Phys. **181** (1996), 205-263.