# Homological and Ring Properties of Formal Power Series Rings

A. V. Mikhalev, C. I. Sonin, A. A. Tuganbaev, and D. A. Tuganbaev

**Abstract.** Some recent results on homological and ring properties of formal power series (including also skew power series rings and skew Laurent series rings) are exposed from a unified point of view. The main lines of investigations: regularity conditions (to be a regular, biregular or Abelian regular ring); hereditary properties (to be a reduced, Rickartian or semihereditary ring); conditions on the lattice of ideals (to be a Bezout or distributive ring).

**Résumé.** Des résultats récent sur des propriétés d'homologie et d'anneau de séries formelles (incluant les anneaux de séries formelles gauches et les anneaux de séries gauche de Laurent) sont présentés d'un point de vue unifié. Les principales directions de recherche sont: les conditions de régularité (régulier, birégulier ou anneau Abelien régulier); les propriétés héréditaires (réduit, anneau Rickartien ou semi-héréditaire); les conditions sur le treillis des idéaux (un treillis distributif ou de Besout).

## 1 General Results

**1.1** Let $\varphi$ be an injective endomorphism of a ring $A$. We denote by $A_\ell[[x,\varphi]]$ the *left skew (power) series ring* consisting of formal series $\sum_{i=0}^{\infty} a_i x^i$ of the variable $x$ with canonical coefficients $a_i \in A$, where addition is defined naturally and multiplication is defined by the rule $x^i a = \varphi^i(a)x^i$.

The *right skew (power) series ring* $A_r[[x,\varphi]]$ consists of series $\sum_{i=0}^{\infty} x^i a_i$, and their multiplication is defined by the rule $ax^i = x^i\varphi^i(a)$.

The *left skew polynomial ring* $A_\ell[x,\varphi] \subset A_\ell[[x,\varphi]]$ and *right skew polynomial ring* $A_r[x,\varphi] \subset A_r[[x,\varphi]]$ are the subrings of skew power series rings $A_\ell[[x,\varphi]]$ and $A_r[[x,\varphi]]$, respectively, consisting of the series with a finite number of nonzero coefficients.

Let $\varphi$ be an automorphism of a ring $A$. Analogously, we define the *left skew Laurent series ring* $A_\ell((x,\varphi))$ and the *right skew Laurent series ring* $A_r((x,\varphi))$ consisting of the series $f \equiv \sum_{i=m}^{\infty} a_i x^i$ and $g \equiv \sum_{i=n}^{\infty} x^i a_i$, where $m = m(f), n = n(g)$ are (maybe, negative) integers, and $x^i a = \varphi^i(a)x^i$ in the left-side case, $ax^i = x^i\varphi^i(a)$ in the right-side case. It follows from the two last equalities that the set $T \equiv \{x^i\}_{i=0}^{\infty}$ is a right and left denominator set both in the ring $A_\ell[[x,\varphi]]$ and in $A_r[[x,\varphi]]$, and isomorphisms $(A_\ell[[x,\varphi]])_T \cong A_\ell((x,\varphi))$, $(A_r[[x,\varphi]])_T \cong A_r((x,\varphi))$ are directly verified.

The *left skew Laurent polynomial ring* $A_\ell[x,x^{-1},\varphi] \subset A_\ell((x,\varphi))$ and the *right skew Laurent polynomial ring* $A_r[x,x^{-1},\varphi] \subset A_r((x,\varphi))$ are the subrings of skew Laurent series rings $A_\ell((x,\varphi))$ and $A_r((x,\varphi))$, respectively, consisting of the series with a finite number of nonzero coefficients.

It follows from the equalities $x^i a = \varphi^i(a)x$ (for $A_\ell((x,\varphi))$) and $ax^i = x\varphi^i(a)$ (for $A_r((x,\varphi))$) that the set $T \equiv \{x^i\}_{i=0}^{\infty}$ is a two-sided Ore subset both in the rings $A_\ell[x,x^{-1},\varphi]$ and $A_r[x,x^{-1},\varphi]$, and isomorphisms $(A_\ell[x,\varphi])_T \cong A_\ell[x,x^{-1},\varphi]$, $(A_r[x,\varphi])_T \cong A_r[x,x^{-1},\varphi]$, $(A_\ell[[x,\varphi]])_T \cong A_\ell((x,\varphi))$, $(A_r[[x,\varphi]])_T \cong A_r((x,\varphi))$ are directly verified.

If the series $f$ belongs to one of the rings $A_\ell[[x,\varphi]]$, $A_r[[x,\varphi]]$, $A_\ell((x,\varphi))$, $A_r((x,\varphi))$, then we denote by $f_i$ and by $C(f)$ the coefficient $a_i \in A$ of $x^i$ in the canonical form of $f$ and the *content* of $f$, (i.e., the ideal of $A$ generated by all coefficients $f_i$), respectively. If a polynomial $g$ belongs to one of the rings $A_\ell[x,\varphi]$, $A_r[x,\varphi]$, $A_\ell[x,x^{-1},\varphi]$, $A_r[x,x^{-1},\varphi]$, then we denote by $\deg(g)$ the *degree* of $g$.

**1.2** Let $\varphi$ be an injective endomorphism of a ring $A$, and let $R \equiv A_\ell[x,\varphi]$. Then the following assertions hold.

(1) If $A$ is a domain, then the rings $R$, $A_r[x,\varphi]$, $A_\ell[[x,\varphi]]$, and $A_r[[x,\varphi]]$ are domains.

(2) If for any $a \in A$, the right ideal $(a+x)R$ of $R$ is an ideal of this ring, then $\varphi$ is the identity automorphism, and $A$ is commutative.

(3) If $A$ is a division ring, and one of the rings $R$, $A_\ell[[x,\varphi]]$ is right uniform, then $\varphi$ is an automorphism, and $R$ is a right and left principal ideal domain.

**1.3** Let $\varphi$ be an automorphism of a ring $A$. For every subset $B$ of the ring $A$, let us denote by $\overline{B}$ the subset of the ring $A_\ell((x,\varphi))$ generated by all the series whose coefficients are contained in $B$. Then the following assertions hold.

(1) If $B$ and $C$ are right ideals of the ring $A$ and $B$ properly contains $C$, then the right ideal $\overline{B}$ of the ring $A_\ell((x,\varphi))$ properly contains $\overline{C}$.

(2) If $B$ is an ideal in $A$ and $B^n = 0$, then $\overline{B}^n = 0$.

(3) $A$ is a domain $\Longleftrightarrow$
$A_\ell((x,\varphi))$ is a domain.

(4) If $A_\ell((x,\varphi))$ is a semiprime ring, then $A$ is a semiprime ring.

(5) If $B$ is a minimal right ideal of the ring $A$, then $\overline{B}$ is a minimal right ideal of the ring $A_\ell((x,\varphi))$.

(6) If the ring $A_\ell((x,\varphi))$ is right Artinian (right Noetherian), then the ring $A$ is right Artinian (right Noetherian).

(7) $A_\ell((x,\varphi))$ is a semisimple Artinian ring $\Longleftrightarrow$
$A$ is a semisimple Artinian ring.

(8) $A$ is a division ring $\Longleftrightarrow$
$A_\ell((x,\varphi))$ is a division ring.

**1.4** Let $\varphi$ be an automorphism of a ring $A$. If $B$ is a subset of the ring $A$, then $\overline{B}$ denotes the subset of the ring $A_\ell((x,\varphi))$ generated by all the series whose coefficients are contained in $B$.

A right (left, two-sided) ideal $B$ of the ring $A$ is said to be *$\varphi$-invariant* if $\varphi(B) = B$.

A ring $A$ is said to be *$\varphi$-prime* if $BC \neq 0$ for all nonzero $\varphi$-invariant ideals $B$ and $C$ of the ring $A$.

A ring $A$ is said to be *$\varphi$-primitive* if $A$ has a maximal right ideal which does not contain nonzero $\varphi$-invariant ideals of the ring $A$.

A ring $A$ is said to be *$\varphi$-reduced* if $a\varphi(a) \neq 0$ for every nonzero element $a$ of the ring $A$.

Let $\varphi$ be an automorphism of a ring $A$. Then the following assertions hold.

(1) If $B$ is a proper $\varphi$-ideal of the ring $A$, then $\overline{B}$ is a proper ideal of the ring $A_\ell((x, \varphi))$.

(2) Let $f = \sum_{i=m}^{\infty} f_i x^i \in A_\ell((x, \varphi))$, where $(f_i \in A)$ and $f_m$ is an invertible element of the ring $A$.

Then the Laurent series $f$ is invertible in the ring $A_\ell((x, \varphi))$.

(3) $A_\ell((x, \varphi))$ is a simple ring $\Longleftrightarrow$
$A$ has no nonzero proper $\varphi$-ideals.

**1.5** Constructibility of the rational closure of the group ring of a linearly ordered group in the Malcev skew field of series was shown in [18].

# 2 Regular, Biregular, and Abelian Regular Rings

**2.1 Normal, quasi-invariant, and invariant rings.** All rings are assumed to be associative and to have a nonzero identity element.

A ring is *normal* if all its idempotents are central.

A module $M$ is *quasi-invariant (invariant)* if all its maximal submodules (all its submodules) are fully invariant in $M$.

A ring $A$ is right quasi-invariant (right invariant) $\Longleftrightarrow$
all maximal right ideals (all right ideals) of $A$ are ideals $\Longleftrightarrow$
each cyclic right $A$-module is quasi-invariant (invariant).

**2.2** A ring $A$ is said to be *regular* if for any $a \in A$, there exists $b \in A$ such that $a = aba$.

A ring is *biregular* if every its 1-generated two-sided ideal is generated by a central idempotent.

An *Abelian regular* ring is any ring $A$ which satisfies the following equivalent conditions.

(1) For any $a \in A$, there exists $b \in A$ such that $a = a^2 b$.

(2) For any $a \in A$, there exists $b \in A$ such that $a = ba^2$.

(3) Every element of $A$ is a product of a central idempotent and a unit.

(4) $A$ is a regular reduced ring.

(5) $A$ is a regular normal ring.

(6) $A$ is a regular invariant ring.

**2.3 ([50])** Let $\varphi$ be an automorphism of the ring $A$ such that $\varphi^n \equiv 1$ for some positive integer $n$. Then the following conditions are equivalent.

(1) $A_\ell((x,\varphi))$ is a regular ring.
(2) $A_\ell((x,\varphi))$ is a unit-regular ring.
(3) $A_\ell((x,\varphi))$ is a semisimple Artinian ring.
(4) $A$ is a semisimple Artinian ring.

**2.4 ([51])** Let $\varphi$ be an automorphism of the ring $A$. Then the following conditions are equivalent.
(1) $A_\ell((x,\varphi))$ is a biregular ring.
(2) $A_\ell((x,\varphi))$ is a finite direct product of simple rings $R_1,\dots R_n$.
(3) $A$ is a finite direct product rings $A_1,\dots,A_n$ with the identity elements $e_1,\dots,e_n$, $\varphi(e_i)=e_i$ for all $i$, and every ring $A_i$ coincides with any its nonzero $\varphi$-invariant ideal.

**2.5 ([59])** Let $\varphi$ be an automorphism of an Abelian regular ring $A$ such that $\varphi(e)=e$ for every central idempotent $\varphi$ of the ring $A$.

Then $A$ is a $\varphi$-reduced ring, $A_\ell((x,\varphi))$ is a reduced ring, all idempotents of the ring $A_\ell((x,\varphi))$ are central and are contained in $A$, and every nonzero right ideal of the ring $R$ contains a nonzero central idempotent.

**2.6 ([50], [53])** Let $\varphi$ be an automorphism of a ring $A$, and let $R\equiv A_\ell((x,\varphi))$. Then the following conditions are equivalent.
(1) $R$ is an Abelian regular ring.
(2) $R$ is a finite direct product of division rings.
(3) $A$ is a $\varphi$-reduced ring, and $R/J(R)$ is a regular ring.
(4) $A$ is a finite direct product of division rings and $\varphi(e)=e$ for every central idempotent $e$ of the ring $A$.

## 3 Reduced, Rickartian, and Semihereditary Rings

**3.1 ([60]) Reduced rings.** A ring which has no nonzero nilpotent elements is called a *reduced* ring.

Let $\varphi$ be an injective endomorphism of a ring $A$. The following conditions are equivalent.
(1) The ring $A_\ell[[x,\varphi]]$ is reduced.
(2) $A_\ell[x,\varphi]$ is reduced.
(3) $A_r[[x,\varphi]]$ is reduced.
(4) $A_r[x,\varphi]$ is reduced.
(5) $A$ is reduced, and $a\varphi^n(a)\neq 0$ for any nonzero $a\in A$ and for any $n\geq 0$.
(6) $A$ is reduced, and $\varphi^n(a)a\neq 0$ for any nonzero $a\in A$ and for any $n\geq 0$.

**3.2 ([53], [60])** Let $\varphi$ be an automorphism of a ring $A$. The following conditions are equivalent.
(1) The ring $A_\ell((x,\varphi))$ is reduced.

(2) $A_\ell[x, x^{-1}, \varphi]$ is reduced.

(3) $A_r((x, \varphi))$ is reduced.

(4) $A_r[x, x^{-1}, \varphi]$ is reduced.

(5) $A$ is reduced, and $a\varphi^n(a) \neq 0$ for any nonzero $a \in A$ and for any integer $n$.

(6) $A$ is reduced, and $\varphi^n(a)a \neq 0$ for any nonzero $a \in A$ and any integer $n$.

**3.3 Rickartian, semihereditary and flat modules.** A module $M$ is *Rickartian* (*resp. semihereditary*) if all cyclic submodules (resp. all cyclic submodules) of $M$ are projective.

A module $E_A$ is *flat* if for any monomorphism of left $A$-modules $u : M_1 \to M_2$, the group homomorphism $E \otimes M_1 \to E \otimes M_2$ is a monomorphism.

**3.4 ([60])** Let $\varphi$ be an injective endomorphism of a reduced ring $A$ such that $a\varphi^n(a) \neq 0$ for any nonzero $a \in A$ and each $n \geq 0$. The following conditions are equivalent.

(1) $A_\ell[x, \varphi]$ is right Rickartian.

(2) $A_\ell[x, \varphi]$ is left Rickartian.

(3) $A_r[x, \varphi]$ is right Rickartian.

(4) $A_r[x, \varphi]$ is left Rickartian.

(5) The annihilator of any finitely generated ideal in the reduced ring $A$ is generated by a central idempotent.

(6) $A$ is right or left Rickartian.

**3.5 ([60])** Let $\varphi$ be an injective endomorphism of a reduced ring $A$ such that $a\varphi^n(a) \neq 0$ for any nonzero $a \in A$ and each $n \geq 0$. Then the following conditions are equivalent.

(1) $A_\ell[[x, \varphi]]$ is right Rickartian.

(2) $A_\ell[[x, \varphi]]$ is left Rickartian.

(3) $S \equiv A_r[[x, \varphi]]$ is right Rickartian.

(4) $S \equiv A_r[[x, \varphi]]$ is left Rickartian.

(5) The annihilator of any countably generated ideal in a reduced ring $A$ is generated by a central idempotent.

**3.6 ([54])** Let $\varphi$ be an automorphism of a ring $A$, and let all 2-generated right ideals of $A_\ell[[x, \varphi]]$ be flat.

Then the ring $A$ is regular.

**3.7 ([60])** A module $M_A$ is *countably injective* if every homomorphism $B_A \to M$, where $B$ is an arbitrary countably generated right ideal of $A$, can be extended to a homomorphism $A_A \to M$.

(1) There exists a commutative regular countably injective ring $D$ such that $D$ is not self-injective, and $D$ is a factor ring of a commutative regular self-injective ring $A$.

(2) There exists a commutative regular ring $D$ such that $D$ is a factor ring of a commutative regular self-injective ring $A$, $D$ has a countably generated ideal $B$ such that $r_D(B)$ is not generated (as an ideal) by an idempotent, and $D$ is not self-injective.

**3.8** (**[56]**) Let $\varphi$ be an automorphism of a normal ring $A$ such that $\varphi(e) = e$ for any idempotent $e \in A$. The following conditions are equivalent.

(1) All submodules of flat $A_\ell[[x, \varphi]]$-modules are flat.

(2) All 2-generated right ideals of $A_\ell[[x, \varphi]]$ are flat.

(3) All 2-generated left ideals of $A_\ell[[x, \varphi]]$ are flat.

(4) $A$ is a Abelian regular countably injective ring.

**3.9** (**[56]**) Let $\varphi$ be an automorphism of a normal ring $A$ such that $\varphi(e) = e$ for any idempotent $e \in A$. The following conditions are equivalent.

(1) The ring $A_\ell[[x, \varphi]]$ is right semihereditary.

(2) $A_\ell[[x, \varphi]]$ is left semihereditary.

(3) All 2-generated right ideals of $A_\ell[[x, \varphi]]$ are projective.

(4) All 2-generated left ideals of $A_\ell[[x, \varphi]]$ are projective.

(5) $A$ is a Abelian regular countably injective ring, and the annihilator of any countably generated ideal in $A$ is generated by a central idempotent.

**3.10** (**[60]**) There exists a commutative regular countably injective ring $D$ such that $D[[x]]$ is a commutative distributive reduced Bezout ring, all submodules of flat $D[[x]]$-modules are flat, $D[[x]]$ is not a Rickartian ring, and a classical ring of quotients of $D[[x]]$ is not regular.

## 4 Bezout and Distributive Rings

**4.1 Distributive, uniserial and Bezout modules.** A module $M$ is said to be *distributive* if the lattice $\mathrm{Lat}(M)$ of all its submodules is distributive, i.e., $F\bigcap(G+H) = F\bigcap G + F\bigcap H$ for all submodules $F$, $G$, and $H$ of the module $M$.

A module $M$ is said to be *uniserial* if any two submodules of the module $M$ are comparable with respect to inclusion.

A module $M$ is called a *Bezout* module or a *locally cyclic* module if every finitely generated submodule of $M$ is cyclic.

**4.2** (**[58]**) Let $\varphi$ be an injective endomorphism of a ring $A$. The following conditions are equivalent.

(1) $A_\ell[x, \varphi]$ is a right Bezout ring, and either $A$ is right quasi-invariant or all annihilator right ideals of $A$ are ideals.

(2) $A$ is Abelian regular, $\varphi$ is an automorphism, and $A_\ell[x, x^{-1}, \varphi]$, $A_\ell[x, \varphi]$ are semihereditary reduced Bezout rings.

(3) $A$ is Abelian regular, $\varphi$ is an automorphism, for any idempotent $e \in A$, the equality $\varphi(e) = e$ holds, and $e$ is central in $A_\ell[[x, \varphi]]$.

**4.3 ([60])** For a ring $A$, the following conditions are equivalent.

(1) $A[x, x^{-1}]$ is a right distributive ring.

(2) $A[x, x^{-1}]$ is a right quasi-invariant right Bezout ring.

(3) $A[x, x^{-1}]$ is a commutative distributive Bezout ring.

(4) $A$ is a commutative regular ring.

**4.4 ([60])** Let $\varphi$ be an injective endomorphism of a ring $A$. Then the following conditions are equivalent.

(1) $A_\ell[x, \varphi]$ is a right distributive ring.

(2) $A$ is a commutative regular ring, and $\varphi \equiv 1$.

**4.5 ([54], [55], [57], and [58])** Let $\varphi$ be an injective endomorphism of a ring $A$. The following conditions are equivalent.

(1) $A_\ell[[x, \varphi]]$ is a right distributive ring.

(2) $A_\ell[[x, \varphi]]$ is a right Bezout ring, and either $A$ is right quasi-invariant, or right annihilators of all elements in $A$ are ideals.

(3) $A_\ell[[x, \varphi]]$ is either a right distributive ring or a right Bezout ring, and $A$ is Abelian regular.

(4) $A_\ell[[x, \varphi]]$ is a distributive reduced Bezout ring, and all submodules of flat $A$-modules are flat.

(5) $A$ is a Abelian regular countably injective ring, $\varphi$ is an automorphism, and $\varphi(e) = e$ for any idempotent $e \in A$.

**4.6 ([55])** (1) There exists a commutative regular ring $A$ such that $A$ is not countably injective, and the ring $A[[x]]$ is not right or left distributive.

Let $T$ be a field, and let $A$ be the ring formed by all eventually constant sequences $f = (f_n)_{n=0}^{\infty}$ of elements of $T$ (for each $(f_n) \in A$, there exists a number $N = N(f)$ such that $f_n = f_N$ for all $n \geq N$). Then $A$ is the requied ring.

(2) If $A$ is the division ring of real quaternions, then the ring $A[[x]]$ is distributive, and the ring $A[x]$ is not right or left distributive.

If $A$ is the commutative regular ring constructed in (1), then $A[x]$ is distributive, and $A[[x]]$ is not right or left distributive.

**4.7 ([58], [60])** Assume that a ring $A$ has an injective endomorphism $\varphi$ such that $\varphi(a)$ is a unit of $A$ for any nonzero $a \in A$. Set $R \equiv A_r[[x, \varphi]]$. Then the following assertions hold.

(1) $A$ and $R$ are domains.

(2) $x^n R \subseteq ax^n R$ and $(a + xf)R = aR$ for any nonzero $a \in A$, for each positive integer $n$, and for any series $f \in R$.

(3) Let $M$ and $N$ be nonzero principal right ideals of $R$.

Then there exist two nonzero principal right ideals $D$ and $E$ of $A$ and nonnegative integers $m, n$ such that $M = x^m DR$, $N = x^n ER$, and the equality $M = N$ is equivalent to the equalities $m = n$ and $D = E$.

In addition, $M \subseteq N \iff$ either $m < n$ or $m = n, D \subseteq E$.

(4) If $A$ is right uniserial (resp. right distributive, a ring with the maximum condition on principal right ideals), then $R$ is right uniserial (resp. right distributive, a ring with the maximum condition on principal right ideals).

(5) If $A$ is a right uniserial right Noetherian domain, then $R$ is a right uniserial right Noetherian principal right ideal domain.

(6) If $a$ is a nonzero noninvertible element of the domain $A$, then $Ra \bigcap Rx = 0$.

**4.8 ([58], [60])** There exists a commutative uniserial principal ideal domain $A$ which is not a field, and $A$ has an injective ring endomorphism $\varphi$ such that $\varphi(a) \in U(A)$ for all $a \in A \setminus 0$.

In addition, the ring $A_r[[x, \varphi]] \equiv R$ possesses the following properties.

(1) $R$ is a right uniserial principal right ideal domain, and therefore, $R$ is a right distributive right Noetherian right hereditary right Bezout domain.

(2) $R$ is not left distributive, $R$ is not left finite-dimensional, and $R$ is not a left Bezout ring.

(3) There exist two nonzero distinct completely prime ideals $M$ and $N$ of $R$ such that $N = MN \subset M \subseteq J(R)$, $N \neq NM \subset N$, and $R/NM$ is a right uniserial right Noetherian principal right ideal ring which contains a nonzero nonmaximal nilpotent completely prime ideal $N/NM$.

(4) Multiplication of completely prime ideals of the right uniserial principal right ideal domain $R$ is not commutative.

In addition, there exists a completely prime ideal $N$ such that $N$ is not a finitely generated left ideal, and $R$ has an indecomposable right uniserial right Noetherian factor ring $\widehat{R}$ which is neither right Artinian nor semiprime.

**4.9 ([58], [60])** For an injective endomorphism $\varphi$ of a ring $A$, the following assertions hold.

(1) Let $A$ be an indecomposable ring $A$. Then
$A_r[[x, \varphi]]$ is a right distributive ring $\iff$
$A$ is a right distributive domain, and $\varphi(a) \in U(A)$ for any nonzero $a \in A$.

(2) $A_r[[x, \varphi]]$ is a right uniserial ring $\iff$
$A$ is right uniserial, and $\varphi(a)$ is a unit of $A$ for any nonzero $a \in A$.

**4.10 ([61], [62])** Let $\varphi$ be an automorphism of a ring $A$. Then the following conditions are equivalent.

(1) $A_\ell((x, \varphi))$ is a right uniserial ring.

(2) $A_\ell((x, \varphi))$ is a right uniserial right Artinian ring.

(3) $A$ is a right uniserial right Artinian ring.

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